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Abstract: *This paper presents the initial efforts by the authors to introduce uncertainty in the stress analysis of reinforced concrete flexural members. A singly reinforced concrete beam subjected to an interval load is taken up for analysis. Using extension principle, the internal moment of resistance of the beam is expressed as a function of interval values of stresses in concrete and steel. The stress distribution model for the cross section of the beam is modified for this purpose. The internal moment of resistance is then equated to the external bending moment due to interval loads acting on the beam. The stresses in concrete and steel are obtained as interval values. The sensitivity of stresses in steel and concrete to corresponding variation of interval values of load about its mean values is explored.*

AMS subject classification: 74S05, 65G20, 65G40, 65M60

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Correspondence
Dr. M.V.Rama Rao
Associate Professor, Department of Civil Engineering
Vasavi College of Engineering, Hyderabad-500 031, INDIA
dr.mvrr@gmail.com
Dr. Andrzej Pownuk
Department of Mathematical Sciences, The University of Texas at El Paso
500 West University Avenue, El Paso, Texas 79968-0514, USA
ampownuk@utep.edu

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The University of Texas at El Paso
Department of Mathematical Sciences
500 West University, El Paso, TX 79968
Email: mathdept@math.utep.edu
URL: http://www.math.utep.edu
Phone: 1.915.747.5761
Fax: 1.915.747.6502

1 Introduction

Analysis of rectangular beams of reinforced concrete is based on nonlinear and/or discontinuous stress-strain relationships and such analyses are difficult to perform. Provided the nature of loading, the beam dimensions, the materials used and the quantity of reinforcement are known, the theory of reinforced concrete permits the analysis of stresses, strains, deflections, crack spacing and width and also the collapse load. Further, the aim of analyzing the beam is to locate the neutral axis depth, find out the stresses in compression concrete and tensile reinforcement and also compute the moment of resistance. The aim of the designer of reinforced concrete beams is to predict the entire spectrum of behavior in mathematical terms, identify the parameters which influence this behavior, and obtain the cracking, deflection and collapse limit loads. There are usually innumerable answers to a design problem. Thus the design is followed by analysis and a final selection is obtained by a process of iteration. Thus the design process becomes clear only when the process of analysis is learnt thoroughly.

In the traditional (deterministic) methods of analysis, all the parameters of the system are taken to be precisely known. In practice, however, there is always some degree of uncertainty associated with the actual values for structural parameters. As a consequence of this, the structural system will always exhibit some degree of uncertainty. A reliable approach to handle uncertainty in a structural system is the use of interval algebra. In this approach, uncertainties in structural parameters will be introduced as interval values i.e., the values are known to lie between two limits, but the exact values are unknown. Thus, the problem is of determining conservative intervals for the structural response. Though interval arithmetic was introduced by Moore [9], the application of interval concepts to structural analysis is more recent. Modeling with intervals provides a link between design and analysis where uncertainty may be represented by bounded sets of parameters. Interval computation has become a significant computing tool with the software packages developed in the past decade. In the present work, interval algebra is used to predict the stress distribution in a reinforced concrete beam subjected to an interval moment.

2 Literature survey

In the literature there are several methods for solution of equations with interval parameters. In the year 1966, Moore [9] discussed the problem of solution of system of linear interval equations. There are many methods of solution of such equation. Many of them are discussed in the book [11] by Neumaier. Neumaier and Pownuk [12] explored properties of positive definite interval matrices. Their algorithm works even for very large uncertainty in parameters. In their work Köylüoglu, Cakmak, Nielsen [7] applied the concept of interval matrix to solution of FEM equations with uncertain parameters. System of linear interval equation with dependent parameters and symmetric matrix was discussed by Jansson [6]. Muhanna and Mullen [10] handled uncertainty in mechanics problems on using an interval-based approach. Muhanna's algorithm is modified by Rama Rao [15] to study the cumulative effect of multiple uncertainties on the structural response.

Skalna, Rama Rao and Pownuk [18] investigated the solution of systems of fuzzy equations in structural mechanics. Ben-Haim and Elishakoff [2] introduced ellipsoid uncertainty. Akpan *et. al* [13] used response surface method in order to approximate fuzzy solution. Vertex solution methodology that was based on α-cut representation was used for the fuzzy analysis. McWilliam [8] described several method of solution of interval equations. Rao and Chen [16] developed a new search-based algorithm to solve a system of linear interval equations to account for uncertainties in engineering problems. The algorithm performs search operations with an accelerated step size in order to locate the optimal setting of the hull of the solution.

Several models were proposed to describe the stress distribution in the cross section of a concrete beam subjected to pure flexure. Initially, the parabolic model was proposed by Hognestad [5] in 1951. This was followed by an exponential model proposed by Smith and Young [19] and Desai and Krishnan model [3]. These models are applicable to concretes with strength below 40 MPa. The Indian standard code of practice for plain and reinforced concrete IS 456-2000 [1] allows the assumption of any suitable relationship between the compressive stress distribution in concrete and the strain in concrete i.e. rectangle, trapezoid, parabola or any other shape which results in prediction of strength in substantial agreement with the results of test. The stress distribution model suggested by the Indian code IS 456-2000 is followed in the present study (Fig.1).

3 Stress analysis of a singly reinforced concrete section

3.1 Stress distribution due to a crisp moment

A singly reinforced concrete section shown in Fig.1 with is taken up for analysis of stresses and strains in concrete and steel. The beam has a width of b and an effective depth of d. The beam is subjected to a maximum external moment M. Strain-distribution is linear and ε_{cc} is the strain in concrete at the extreme compression fiber and $\varepsilon_s$is the strain in steel. Let x be the neutral axis depth from the extreme compression fiber. The aim of analyzing the beam is to locate this neutral axis depth, find out the stresses in compression concrete and the tensile reinforcement and also compute the moment of resistance. The stress-distribution is concrete is parabolic and concrete in tension is neglected. The strainε_{cy} at any level y below the neutral axis (y$\leqslant$ x) is

$$\varepsilon_{cy} = \left(\frac{y}{x}\right)\varepsilon_{cc} \tag{1}$$

The corresponding stress f_{cy} is

$$f\text{cy} = f\text{co}\left[2\left(\frac{\varepsilon_{cy}}{\varepsilon_{co}}\right) - \left(\frac{\varepsilon_{c}}{\varepsilon_{co}}\right)^{2}\right] \tag{2}$$

for $\varepsilon\text{cy} \leqslant \varepsilon\text{co}$ and $f\text{cy} = f\text{co}$ for $\varepsilon\text{cy} = \varepsilon\text{co}$.
Total compressive force in concrete N_c is given by

$$N_c = \int_{y=0}^{y=x} f_{cy} b dy = \left[C_1\varepsilon_{cc} - C_1\varepsilon_{cc}^2\right] x \tag{3}$$

where

$$C_1 = \left(\frac{bf_{co}}{\varepsilon_{co}}\right) \ \ and \ \ C_2 = \left(\frac{bf_{co}}{3\varepsilon_{co}^2}\right). \tag{4}$$

Tensile stress in steel

$$N_s = (A_s E_s)\varepsilon_{cc}\left(\frac{d-x}{x}\right) \tag{5}$$

If there are no external loads, the equation of longitudinal equilibrium, $N_s = N_c$ leads to the quadratic equation

$$\left[C_1 - C_2\varepsilon_{cc}\right] x^2 + A_s E_s x - A_s E_s d = 0 \tag{6}$$

Depth of resultant compressive force from the neutral axis $\overline{y}$ is given by

$$\overline{y} = \frac{\int_{y=0}^{y=x} b f_{cy} y dy}{\int_{y=0}^{y=x} b f_{cy} dy} = \frac{\left[\left(\frac{2C_1}{3}\right) - \left(\frac{3C_2}{4}\right)\varepsilon_{cc}\right]}{\left[C_1 - C_2\varepsilon_{cc}\right]} x \tag{7}$$

Internal resisting moment M_R is given by

$$M_R = N_c \times z = N_c \times (\overline{y} + d - x) \tag{8}$$

For equilibrium the external moment M is equated to the internal moment of resistance M_R as

$$M \leqslant M_R \tag{9}$$

The neutral axis depth x can determined by solving Eq. (6) only when ε_{cc} is known. Thus a trial and error procedure is adopted where in ε_{cc} is assumed and the corresponding values of N_c ,$\overline{y}$ and internal resisting moment M_R are obtained using Eq. (7) and Eq. (8) such that Eq. (9) is satisfied.
Stress in steel

$$f_s = E_s \varepsilon_s = E_s \left(\frac{d-x}{x} \right) \varepsilon_{cc} \leqslant 0.87 f_y \tag{10}$$

Total tensile force in steel reinforcement

$$N_s = A_s f_s \tag{11}$$

3.2 Stress distribution due to an uncertain moment

Consider the case of a singly reinforced concrete beam subjected to an uncertain interval moment $M = \left[\underline{M}, \overline{M}\right]$. The uncertainty in external moment arises out of uncertainty of loads acting on the beam. Correspondingly the resulting stresses and strains in concrete and steel are also uncertain and are modeled using interval numbers. Using extension principle [20] all the equations developed in the previous section can be extended and made applicable to the interval case. The objective of the present study is to determine distribution of stresses and strain across the cross section of the beam. Two new approaches have been proposed for this purpose: a search based algorithm and a procedure based on Pownuk's sensitivity analysis [14]. These methods are outlined as follows:

Search-based algorithm (SBA)

A search based algorithm (SBA) is developed to perform search operations with an accelerated step size in order to compute the optimal setting for the interval value of strain in concrete is $\varepsilon_{cc} = [\underline{\varepsilon}, \overline{\varepsilon}]$. The algorithm is outlined below:

Algorithm-1

1. The mid-value M of the given interval moment M is computed as $M = \frac{\underline{M}+\overline{M}}{2}$.
2. Various values of ε_{cc} are assumed and the neutral axis depth x and the corresponding values of $N_c, \overline{y}$ and M_R are determined by using a trial and error procedure outlined in the previous section.
3. The interval strain in concrete ε_{cc} is initially approximated as the point interval $[\varepsilon_{cc}, \varepsilon_{cc}]$.

4. The lower and upper bounds of ε_{cc} are obtained as

$$\varepsilon_{cc} = [\varepsilon_{cc} - \lambda_1 d\underline{\varepsilon}, \varepsilon_{cc} + \lambda_2 d\overline{\varepsilon}] \tag{12}$$

   where $d\underline{\varepsilon}$ and $d\overline{\varepsilon}$ are the step sizes in strain to obtain the lower and upper bounds, $\lambda_1$and λ_2 being the corresponding multipliers. Initially $\lambda_1$and λ_2 are taken as unity.

5. While both λ_1,λ_2 are non-zero, $d\underline{\varepsilon}$ and $d\overline{\varepsilon}$ are incremented and ε_{cc} is computed.

6. The interval form of the Eq.(6) is solved by the procedure outlined by Hansen and Walster [4].

7. The procedure is continued iteratively till the interval form of Eq.(9) i.e. $M \leqslant M_R$ is satisfied. The computations performed are outlined below:

   (a) The interval values of x,$\overline{y}$,z,N_c and the interval internal resisting moment $\mathbf{M}_R = \left[\underline{M_R}, \overline{M}_R\right]$ are computed. If η is a very small number

   (b) $\lambda_1$is set to zero if

$$\left|\frac{\underline{M_R} - \underline{M}}{\underline{M_R}}\right| \leqslant \eta \tag{13}$$

   (c) λ_2 is set to zero if

$$\left|\frac{\overline{M}_R - \overline{M}}{\overline{M}_R}\right| \leqslant \eta \tag{14}$$

   (d) The search is discontinued when $\lambda_1 = \lambda_2$=0.

4 Sensitivity analysis method

4.1 Extreme values of ε_{cc} and x.

Unknown variables ε_{cc} and x can be found from the system of equation (8) and equilibrium equation $N_s = N_c$. Lets introduce a new notation

$$\begin{cases} F_1 = F_1(\varepsilon_{cc}, x, p_1, ..., p_m) = M_R - N_c \cdot (\overline{y} + d - x) = 0 \\ F_2 = F_2(\varepsilon_{cc}, x, p_1, ..., p_m) = N_s - N_c = 0 \end{cases} \tag{15}$$

where $p_1 = M$, $p_2 = f_{co}$, $p_3 = A_S$, $p_4 = \varepsilon_{co}$, $p_5 = E_S$, $p_6 = b$, $p_7 = d$.

Because the problem is relatively simple and the intervals $\left[p_i^-, p_i^+\right]$ are usually narrow, then it is possible to solve the problem using sensitivity analysis method [14].

Let calculate sensitivity of the solution with respect to the parameter p_i.

$$\frac{\partial}{\partial p_i}F_1 = \frac{\partial F_1}{\partial \varepsilon_{cc}}\frac{\partial \varepsilon_{cc}}{\partial p_i} + \frac{\partial F_1}{\partial x}\frac{\partial x}{\partial p_i} + \frac{\partial F_1}{\partial p_i} = 0 \tag{16}$$

$$\frac{\partial}{\partial p_i}F_2 = \frac{\partial F_2}{\partial \varepsilon_{cc}}\frac{\partial \varepsilon_{cc}}{\partial p_i} + \frac{\partial F_2}{\partial x}\frac{\partial x}{\partial p_i} + \frac{\partial F_2}{\partial p_i} = 0 \tag{17}$$

In matrix form

$$\begin{bmatrix} \frac{\partial F_1}{\partial \varepsilon_{cc}} & \frac{\partial F_1}{\partial x} \\ \frac{\partial F_2}{\partial \varepsilon_{cc}} & \frac{\partial F_2}{\partial x} \end{bmatrix} \begin{bmatrix} \frac{\partial \varepsilon_{cc}}{\partial p_i} \\ \frac{\partial x}{\partial p_i} \end{bmatrix} = \begin{bmatrix} -\frac{\partial F_1}{\partial p_i} \\ -\frac{\partial F_2}{\partial p_i} \end{bmatrix} \tag{18}$$

Using Cramer's rule the solution is given by the following formulas

$$\frac{\partial \varepsilon_{cc}}{\partial p_i} = -\frac{\begin{vmatrix} \frac{\partial F_1}{\partial p_i} & \frac{\partial F_1}{\partial x} \\ \frac{\partial F_2}{\partial p_i} & \frac{\partial F_2}{\partial x} \end{vmatrix}}{\begin{vmatrix} \frac{\partial F_1}{\partial \varepsilon_{cc}} & \frac{\partial F_1}{\partial x} \\ \frac{\partial F_2}{\partial \varepsilon_{cc}} & \frac{\partial F_2}{\partial x} \end{vmatrix}} = -\frac{\frac{\partial(F_1,F_2)}{\partial(p_i,x)}}{\frac{\partial(F_1,F_2)}{\partial(\varepsilon_{cc},x)}}, \quad \frac{\partial x}{\partial p_i} = -\frac{\begin{vmatrix} \frac{\partial F_1}{\partial \varepsilon_{cc}} & \frac{\partial F_1}{\partial p_i} \\ \frac{\partial F_2}{\partial \varepsilon_{cc}} & \frac{\partial F_2}{\partial p_i} \end{vmatrix}}{\begin{vmatrix} \frac{\partial F_1}{\partial \varepsilon_{cc}} & \frac{\partial F_1}{\partial x} \\ \frac{\partial F_2}{\partial \varepsilon_{cc}} & \frac{\partial F_2}{\partial x} \end{vmatrix}} = -\frac{\frac{\partial(F_1,F_2)}{\partial(\varepsilon_{cc},p_i)}}{\frac{\partial(F_1,F_2)}{\partial(\varepsilon_{cc},x)}} \tag{19}$$

If all Jacobians

$$\frac{\partial(F_1,F_2)}{\partial(\varepsilon_{cc},x)}, \quad \frac{\partial(F_1,F_2)}{\partial(p_i,x)}, \quad \frac{\partial(F_1,F_2)}{\partial(\varepsilon_{cc},p_i)} \tag{20}$$

are regular then the derivatives have constant sign and the relations $\varepsilon_{cc} = \varepsilon_{cc}(p_i)$,$x = x(p_i)$ are monotone. All variables p_i belong to know intervals $p_i \in \left[\underline{p}_i, \overline{p}_i\right]$ because of that sign of the Jacobians can be checked using interval global optimization method [14]. If

$$0 \neq \min_{\varepsilon_{cc}\in[\underline{\varepsilon}_{cc},\overline{\varepsilon}_{cc}],x\in[\underline{x},\overline{x}],p_1\in\left[\underline{p}_1,\overline{p}_1\right],\ldots,p_m\in\left[\underline{p}_m,\overline{p}_m\right]} |\Delta(\varepsilon_{cc}, x, p_1, \ldots, p_m)| \tag{21}$$

then the Jacobian Δ is regular, where $\Delta = \frac{\partial(F_1,F_2)}{\partial(\varepsilon_{cc},x)}$, $\Delta = \frac{\partial(F_1,F_2)}{\partial(p_i,x)}$ or $\Delta = \frac{\partial(F_1,F_2)}{\partial(\varepsilon_{cc},p_i)}$. If the sign of the derivatives is constant then extreme values of the solution can be calculated using endpoints of the intervals $\left[\underline{p}_i, \overline{p}_i\right]$ and sensitivity analysis method [14]. The whole algorithm of calculation is the following:

Algorithm-2

1. Calculate mid point of the intervals $p_{i0} = \frac{\underline{p}_i+\overline{p}_i}{2}$.

2. Solve the system of equation (15) and calculate ε_{cc0}, x_0.

3. Calculate sensitivity of the solution $\frac{\partial \varepsilon_{cc}}{\partial p_i}$, $\frac{\partial x}{\partial p_i}$ from the equation (18).

4. If $\frac{\partial \varepsilon_{cc}}{\partial p_i} \geqslant 0$ then $p_i^{\min,\varepsilon_{cc}} = \underline{p}_i$, $p_i^{\max,\varepsilon_{cc}} = \overline{p}_i$,
   if $\frac{\partial \varepsilon_{cc}}{\partial p_i} < 0$ then $p_i^{\min,\varepsilon_{cc}} = \overline{p}_i$, $p_i^{\max,\varepsilon_{cc}} = \underline{p}_i$.

5. If $\frac{\partial x}{\partial p_i} \geqslant 0$ then $p_i^{\min,x} = \underline{p}_i$, $p_i^{\max,x} = \overline{p}_i$,
   if $\frac{\partial x}{\partial p_i} < 0$ then $p_i^{\min,x} = \overline{p}_i$, $p_i^{\max,x} = \underline{p}_i$.

6. Extreme values of ε_{cc},x can be calculated as a solution of the following system of equations.

7. Verification of the results. If the derivatives have the same sign at the endpoints $p_i^{\min,x}, p_i^{\max,x}, p_i^{\min,\varepsilon_{cc}}, p_i^{\max,\varepsilon_{cc}}$ and in the midpoint then the solution is very reliable.

4.2 First order monotonicity test

In order to improve accuracy of the solution one can apply higher order monotonicity test. In order to verify the sign of the derivatives$\frac{\partial \varepsilon_{cc}}{\partial p_i}$, $\frac{\partial x}{\partial p_i}$ it is possible to expand these derivatives using first (or higher) order Taylor series.

$$\frac{\partial \varepsilon_{cc}(p)}{\partial p_i} \approx \frac{\partial \varepsilon_{cc}(p_0)}{\partial p_i} + \sum_j \frac{\partial^2 \varepsilon_{cc}(p_0)}{\partial p_i \partial p_j} (p_j - p_{j0}) \tag{22}$$

$$\frac{\partial x(p)}{\partial p_i} \approx \frac{\partial x(p_0)}{\partial p_i} + \sum_j \frac{\partial^2 x(p_0)}{\partial p_i \partial p_j} (p_j - p_{j0}) \tag{23}$$

The functions$\varepsilon_{cc} = \varepsilon_{cc}(p)$, $x = x(p)$ are monotone if the derivatives have constant sign. If we know the intervals $p_1 \in [\underline{p}_1, \overline{p}_1], p_2 \in [\underline{p}_2, \overline{p}_2], ...p_m \in [\underline{p}_m, \overline{p}_m]$ to which belong all uncertain parameters, then it is possible to verify the sign of the derivative by calculating an interval extension of the formulas (22), (23). If

$$0 \notin \frac{\partial \varepsilon_{cc}(p_0)}{\partial p_i} + \sum_j \frac{\partial^2 \varepsilon_{cc}(p_0)}{\partial p_i \partial p_j} (\mathbf{p}_j - p_{j0}) \tag{24}$$

$$0 \notin \frac{\partial x(p_0)}{\partial p_i} + \sum_j \frac{\partial^2 x(p_0)}{\partial p_i \partial p_j} (\mathbf{p}_j - p_{j0}) \tag{25}$$

then the functions $\varepsilon_{cc} = \varepsilon_{cc}(p)$, $x = x(p)$ should be monotone.
Derivative of the functions (22) and (23) can be calculated in the following way

$$\frac{\partial}{\partial p_k} \left(\frac{\partial \varepsilon_{cc}(p_0)}{\partial p_i} + \sum_j \frac{\partial^2 \varepsilon_{cc}(p_0)}{\partial p_i \partial p_j} (p_j - p_{j0}) \right) = \frac{\partial^2 \varepsilon_{cc}(p_0)}{\partial p_i \partial p_k} \tag{26}$$

$$\frac{\partial}{\partial p_k} \left(\frac{\partial x(p_0)}{\partial p_i} + \sum_j \frac{\partial^2 x(p_0)}{\partial p_i \partial p_j} (p_j - p_{j0}) \right) = \frac{\partial^2 x(p_0)}{\partial p_i \partial p_k} \tag{27}$$

The sign of the derivatives is constant because of that extreme values can be calculated by using the following procedure

$$if \ \frac{\partial^2 \varepsilon_{cc}(p_0)}{\partial p_i \partial p_k} \geqslant 0 \ then \ p_{k,\varepsilon}^{max} = \overline{p}_k, \ p_{k,\varepsilon}^{min} = \underline{p}_k \ else \ p_{k,\varepsilon}^{max} = \underline{p}_k, \ p_{k,\varepsilon}^{min} = \overline{p}_k, \quad (28)$$

$$if \ \frac{\partial^2 x(p_0)}{\partial p_i \partial p_k} \geqslant 0 \ then \ p_{k,x}^{max} = \overline{p}_k, \ p_{k,x}^{min} = \underline{p}_k \ else \ p_{k,x}^{max} = \underline{p}_k, \ p_{k,x}^{min} = \overline{p}_k. \quad (29)$$

Now extreme values of the derivatives can be approximated by using the following formulas

$$\frac{\partial \underline{\varepsilon}_{cc}}{\partial p_i} \approx \frac{\partial \varepsilon_{cc}(p_0)}{\partial p_i} + \sum_j \frac{\partial^2 \varepsilon_{cc}(p_0)}{\partial p_i \partial p_j} \left(p_{j,\varepsilon}^{min} - p_{j0}\right) \quad (30)$$

$$\frac{\partial \overline{\varepsilon}_{cc}}{\partial p_i} \approx \frac{\partial \varepsilon_{cc}(p_0)}{\partial p_i} + \sum_j \frac{\partial^2 \varepsilon_{cc}(p_0)}{\partial p_i \partial p_j} \left(p_{j,\varepsilon}^{max} - p_{j0}\right) \quad (31)$$

$$\frac{\partial \underline{x}}{\partial p_i} \approx \frac{\partial x(p_0)}{\partial p_i} + \sum_j \frac{\partial^2 x(p_0)}{\partial p_i \partial p_j} \left(p_{j,x}^{min} - p_{j0}\right) \quad (32)$$

$$\frac{\partial \overline{x}}{\partial p_i} \approx \frac{\partial x(p_0)}{\partial p_i} + \sum_j \frac{\partial^2 x(p_0)}{\partial p_i \partial p_j} \left(p_{j,x}^{max} - p_{j0}\right) \quad (33)$$

Now the monotonicity test can be written in the following way

$$If \ \left[\frac{\partial \underline{\varepsilon}_{cc}}{\partial p_k}, \frac{\partial \overline{\varepsilon}_{cc}}{\partial p_k}\right] > 0, \ then \ p_{k,\varepsilon}^{max} = \overline{p}_k, \ p_{k,\varepsilon}^{min} = \underline{p}_k. \quad (34)$$

$$If \ \left[\frac{\partial \underline{\varepsilon}_{cc}}{\partial p_k}, \frac{\partial \overline{\varepsilon}_{cc}}{\partial p_k}\right] < 0, \ then \ p_{k,\varepsilon}^{max} = \underline{p}_k, \ p_{k,\varepsilon}^{min} = \overline{p}_k. \quad (35)$$

$$\underline{\varepsilon}_{cc} \approx \varepsilon_{cc}(p_{1,\varepsilon}^{min}, ..., p_{m,\varepsilon}^{min}), \ \overline{\varepsilon}_{cc} \approx \varepsilon_{cc}(p_{1,\varepsilon}^{max}, ..., p_{m,\varepsilon}^{max}). \quad (36)$$

$$If \ \left[\frac{\partial \underline{x}}{\partial p_k}, \frac{\partial \overline{x}}{\partial p_k}\right] > 0, \ then \ p_{k,x}^{max} = \overline{p}_k, \ p_{k,x}^{min} = \underline{p}_k \quad (37)$$

$$If \ \left[\frac{\partial \underline{x}}{\partial p_k}, \frac{\partial \overline{x}}{\partial p_k}\right] < 0, \ then \ p_{k,x}^{max} = \underline{p}_k, \ p_{k,x}^{min} = \overline{p}_k \quad (38)$$

$$\underline{x} \approx x(p_{1,x}^{min}, ..., p_{m,x}^{min}), \ \overline{x} \approx x(p_{1,x}^{max}, ..., p_{m,x}^{max}). \quad (39)$$

4.3 Second order monotonicity test

In order to improve accuracy of the calculations one can apply higher order monotonicity test.

$$\begin{gathered}\frac{\partial x(p)}{\partial p_i} \approx \frac{\partial x(p_0)}{\partial p_i} + \sum_j \frac{\partial^2 x(p_0)}{\partial p_i \partial p_j}(p_j - p_{j0}) + \\ + \frac{1}{2}\sum_{j,k} \frac{\partial^3 x(p_0)}{\partial p_i \partial p_j \partial p_k}(p_j - p_{j0})(p_k - p_{k0})\end{gathered} \tag{40}$$

Derivatives of the function (57) are the following

$$\begin{gathered}\frac{\partial x(p)}{\partial p_i \partial p_k} \approx \frac{\partial}{\partial p_k}\left(\frac{\partial x(p_0)}{\partial p_i} + \sum_j \frac{\partial^2 x(p_0)}{\partial p_i \partial p_j}(p_j - p_{j0}) + \right. \\ \left. + \frac{1}{2}\sum_{j,k} \frac{\partial^3 x(p_0)}{\partial p_i \partial p_j \partial p_k}(p_j - p_{j0})(p_k - p_{k0})\right) = \\ = \frac{\partial^2 x(p_0)}{\partial p_i \partial p_k} + \sum_j \frac{\partial^3 x(p_0)}{\partial p_i \partial p_j \partial p_k}(p_j - p_{j0})\end{gathered} \tag{41}$$

Next derivative is the following

$$\frac{\partial x(p)}{\partial p_i \partial p_k \partial p_q} \approx \frac{\partial}{\partial p_q}\left(\frac{\partial^2 x(p_0)}{\partial p_i \partial p_k} + \sum_j \frac{\partial^3 x(p_0)}{\partial p_i \partial p_j \partial p_k}(p_j - p_{j0})\right) = \frac{\partial^3 x(p_0)}{\partial p_i \partial p_q \partial p_k} \tag{42}$$

Because the sign of the third derivative is constant then extreme values of second derivative can be calculated using the endpoint of the intervals

$$\begin{gathered} if \;\; \frac{\partial^3 x(p_0)}{\partial p_k \partial p_j \partial p_i} \geqslant 0, \; then \\ p_{k,x}^{max} = \overline{p}_k, \; p_{k,x}^{min} = \underline{p}_k \; else \; p_{k,x}^{max} = \underline{p}_k, \; p_{k,x}^{min} = \overline{p}_k. \end{gathered} \tag{43}$$

$$\frac{\partial \underline{x}}{\partial p_j \partial p_i} \approx \frac{\partial^2 x(p_0)}{\partial p_j \partial p_i} + \sum_k \frac{\partial^3 x(p_0)}{\partial p_k \partial p_j \partial p_i}\left(p_{k,x}^{min} - p_{k0}\right) \tag{44}$$

$$\frac{\partial \overline{x}}{\partial p_j \partial p_i} \approx \frac{\partial^2 x(p_0)}{\partial p_j \partial p_i} + \sum_k \frac{\partial^3 x(p_0)}{\partial p_k \partial p_j \partial p_i}\left(p_{k,x}^{min} - p_{k0}\right) \tag{45}$$

Extreme values of first derivative can be calculated in the following way

$$if \;\; \left[\frac{\partial \underline{x}}{\partial p_j \partial p_i}, \frac{\partial \overline{x}}{\partial p_j \partial p_i}\right] \geqslant 0, \; then \; p_{j,x}^{max} = \overline{p}_j, \; p_{j,x}^{min} = \underline{p}_j \tag{46}$$

$$if \;\; \left[\frac{\partial \underline{x}}{\partial p_j \partial p_i}, \frac{\partial \overline{x}}{\partial p_j \partial p_i}\right] < 0, \; then \; p_{j,x}^{max} = \underline{p}_j, \; p_{j,x}^{min} = \overline{p}_j. \tag{47}$$

$$\begin{gathered}\frac{\partial \underline{x}}{\partial p_i} \approx \frac{\partial x(p_0)}{\partial p_i} + \sum_j \frac{\partial^2 x(p_0)}{\partial p_j \partial p_i}\left(p_{j,x}^{min} - p_{j0}\right) + \\ + \frac{1}{2}\sum_{j,k} \frac{\partial^3 x(p_0)}{\partial p_k \partial p_j \partial p_i}\left(p_{j,x}^{min} - p_{j0}\right)\left(p_{k,x}^{min} - p_{k0}\right)\end{gathered} \tag{48}$$

$$\frac{\partial \overline{x}}{\partial p_i} \approx \frac{\partial x(p_0)}{\partial p_i} + \sum_j \frac{\partial^2 x(p_0)}{\partial p_j \partial p_i} \left(p_{j,x}^{max} - p_{j0}\right) + \\ + \frac{1}{2} \sum_{j,k} \frac{\partial^3 x(p_0)}{\partial p_k \partial p_j \partial p_i} \left(p_{j,x}^{max} - p_{j0}\right) \left(p_{k,x}^{max} - p_{k0}\right) \tag{49}$$

Extreme values of the function can be calculated in the following way

$$if \quad \left[\frac{\partial \underline{x}}{\partial p_i}, \frac{\partial \overline{x}}{\partial p_i}\right] \geqslant 0, \; then \; p_{i,x}^{max} = \overline{p}_i, \; p_{i,x}^{min} = \underline{p}_i \tag{50}$$

$$if \quad \left[\frac{\partial \underline{x}}{\partial p_i}, \frac{\partial \overline{x}}{\partial p_i}\right] < 0, \; then \; p_{i,x}^{max} = \underline{p}_i, \; p_{i,x}^{min} = \overline{p}_i. \tag{51}$$

$$\underline{x} \approx x(p_1^{min}, p_2^{min}, ..., p_m^{min}) \tag{52}$$

$$\overline{x} \approx x(p_1^{max}, p_2^{max}, ..., p_m^{max}) \tag{53}$$

4.4 Higher order monotonicity tests based on Taylor series

It is possible to approximate the value of the function using higher order Taylor series.

$$\frac{\partial x(p)}{\partial p_i} \approx \frac{\partial x(p_0)}{\partial p_i} + \sum_j \frac{\partial^2 x(p_0)}{\partial p_i \partial p_j} (p_j - p_{j0}) + \\ + \frac{1}{2!} \sum_{j,k} \frac{\partial^3 x(p_0)}{\partial p_k \partial p_j \partial p_i} (p_j - p_{j0}) (p_k - p_{k0}) + \\ + \frac{1}{3!} \sum_{j,k,l} \frac{\partial^4 x(p_0)}{\partial p_l \partial p_k \partial p_j \partial p_i} (p_j - p_{j0}) (p_k - p_{k0}) (p_l - p_{l0}) \tag{54}$$

Forth order derivative is constant because of that the extreme values of third derivative can be calculated by using sign of forth derivative.

$$if \quad \frac{\partial^4 x(p_0)}{\partial p_l \partial p_k \partial p_j \partial p_i} \geqslant 0, \; then \\ p_{l,x}^{max} = \overline{p}_l, \; p_{l,x}^{min} = \underline{p}_l \; else \; p_{l,x}^{max} = \underline{p}_l, \; p_{l,x}^{min} = \overline{p}_l. \tag{55}$$

$$\frac{\partial^3 \underline{x}}{\partial p_k \partial p_j \partial p_i} \approx \frac{\partial^3 x(p_0)}{\partial p_k \partial p_j \partial p_i} + \sum_l \frac{\partial^3 x(p_0)}{\partial p_l \partial p_k \partial p_j \partial p_i} \left(p_{l,x}^{min} - p_{k0}\right) \tag{56}$$

$$\frac{\partial^3 \overline{x}}{\partial p_k \partial p_j \partial p_i} \approx \frac{\partial^3 x(p_0)}{\partial p_k \partial p_j \partial p_i} + \sum_l \frac{\partial^3 x(p_0)}{\partial p_l \partial p_k \partial p_j \partial p_i} \left(p_{l,x}^{max} - p_{k0}\right) \tag{57}$$

Using the sign of the third derivative it is possible to calculate extreme values of second derivatives. If we continue that process it is possible to calculate extreme values of the function $x = x(p)$.
In the same way it is possible to calculate extreme values of the function $x = x(p)$ by using n-th order Taylor series. The test is as accurate as the Taylor expansion.

4.5 Higher order monotonicity tests which are based on exact values

It is also possible to calculate the partial derivatives (only n-th order) in the mid point and then calculate the sign of the $n-1$-th order derivatives

$$if\ \frac{\partial x^n(p_0)}{\partial p_{i_1}\partial p_{i_2}...\partial p_{i_{n-1}}\partial p_{i_n}} \geqslant 0\ \ then\ p_{i_n}^{max}=\overline{p}_{i_n},\ p_{i_n}^{min}=\underline{p}_{i_n} \tag{58}$$

$$if\ \frac{\partial x^n(p_0)}{\partial p_{i_1}\partial p_{i_2}...\partial p_{i_{n-1}}\partial p_{i_n}} < 0\ \ then\ p_{i_n}^{max}=\underline{p}_{i_n},\ p_{i_n}^{min}=\overline{p}_{i_n} \tag{59}$$

$$\frac{\partial \underline{x}^{n-1}}{\partial p_{i_1}\partial p_{i_2}...\partial p_{i_{n-1}}} = \frac{\partial x^{n-1}(p_1^{min},...,p_m^{min})}{\partial p_{i_1}\partial p_{i_2}...\partial p_{i_{n-1}}} \tag{60}$$

$$\frac{\partial \overline{x}^{n-1}}{\partial p_{i_1}\partial p_{i_2}...\partial p_{i_{n-1}}} = \frac{\partial x^{n-1}(p_1^{max},...,p_m^{max})}{\partial p_{i_1}\partial p_{i_2}...\partial p_{i_{n-1}}} \tag{61}$$

If we know the sign of the $n-1$-th derivatives it is possible to calculate the range of the $n-2$ derivative

$$if\ \left[\frac{\partial \underline{x}^{n-1}}{\partial p_{i_1}...\partial p_{i_{n-1}}}, \frac{\partial \overline{x}^{n-1}}{\partial p_{i_1}...\partial p_{i_{n-1}}}\right] \geqslant 0\ \ then\ p_{i_{n-1}}^{max}=\overline{p}_{i_{n-1}},\ p_{i_{n-1}}^{min}=\underline{p}_{i_{n-1}} \tag{62}$$

$$if\ \left[\frac{\partial \underline{x}^{n-1}}{\partial p_{i_1}...\partial p_{i_{n-1}}}, \frac{\partial \overline{x}^{n-1}}{\partial p_{i_1}...\partial p_{i_{n-1}}}\right] < 0\ \ then\ p_{i_{n-1}}^{max}=\underline{p}_{i_{n-1}},\ p_{i_{n-1}}^{min}=\overline{p}_{i_{n-1}} \tag{63}$$

$$\frac{\partial \underline{x}^{n-2}}{\partial p_{i_1}\partial p_{i_2}...\partial p_{i_{n-2}}} = \frac{\partial x^{n-2}(p_1^{min},...,p_m^{min})}{\partial p_{i_1}\partial p_{i_2}...\partial p_{i_{n-2}}} \tag{64}$$

$$\frac{\partial \overline{x}^{n-2}}{\partial p_{i_1}\partial p_{i_2}...\partial p_{i_{n-2}}} = \frac{\partial x^{n-2}(p_1^{max},...,p_m^{max})}{\partial p_{i_1}\partial p_{i_2}...\partial p_{i_{n-2}}} \tag{65}$$

If we continue that process it is possible to calculate the range of the function $x = x(p_1, ..., p_m)$.

4.6 Interval stress in extreme concrete fiber

Sensitivity of stress in extreme concrete fiber f_{cc} can be calculated in the following way

$$\frac{\partial}{\partial p_i} f_{cc} = \frac{\partial f_{cc}}{\partial \varepsilon_{cc}}\frac{\partial \varepsilon_{cc}}{\partial p_i} + \frac{\partial f_{cc}}{\partial x}\frac{\partial x}{\partial p_i} + \frac{\partial f_{cc}}{\partial p_i} \tag{66}$$

where $\frac{\partial \varepsilon_{cc}}{\partial p_i}$ and $\frac{\partial x}{\partial p_i}$ are solution of the equation (18).

If $\frac{\partial f_{cc}}{\partial p_i} \geqslant 0$ then $p_i^{\min,f_{cc}} = \underline{p}_i$, $p_i^{\max,f_{cc}} = \overline{p}_i$, if $\frac{\partial f_{cc}}{\partial p_i} < 0$ then $p_i^{\min,f_{cc}} = \overline{p}_i$, $p_i^{\max,f_{cc}} = \underline{p}_i$.

$$\underline{f}_{cc} = f_{cc}\left(\varepsilon_{cc}^{\min,f_{cc}}, x^{\min,f_{cc}}, p_1^{\min,f_{cc}}, ..., p_m^{\min,f_{cc}}\right), \tag{67}$$

$$\overline{f}_{cc} = f_{cc}\left(\varepsilon_{cc}^{\max,f_{cc}}, x^{\max,f_{cc}}, p_1^{\max,f_{cc}}, ..., p_m^{\max,f_{cc}}\right) \tag{68}$$

In the midpoint sensitivity is equal to

$$\frac{\partial}{\partial M} f_{cc} = \frac{\partial f_{cc}}{\partial \varepsilon_{cc}}\frac{\partial \varepsilon_{cc}}{\partial M} + \frac{\partial f_{cc}}{\partial x}\frac{\partial x}{\partial M} + \frac{\partial f_{cc}}{\partial M} \tag{69}$$

Extreme values of stress in extreme concrete fiber calculated form the formulas (67) and (68).
It is possible to use methods which are based on higher order derivatives in the same way as in the section 4.4 or 4.5.

4.7 Interval stress in steel

Sensitivity of stress in steel f_s can be calculated in the following way

$$\frac{\partial}{\partial p_i} f_s = \frac{\partial f_s}{\partial \varepsilon_{cc}}\frac{\partial \varepsilon_{cc}}{\partial p_i} + \frac{\partial f_s}{\partial x}\frac{\partial x}{\partial p_i} + \frac{\partial f_s}{\partial p_i} \tag{70}$$

where $\frac{\partial \varepsilon_{cc}}{\partial p_i}$ and $\frac{\partial x}{\partial p_i}$ are solution of the equation (18).

If $\frac{\partial f_s}{\partial p_i} \geqslant 0$ then $p_i^{\min,f_s} = \underline{p}_i$, $p_i^{\max,f_s} = \overline{p}_i$, if $\frac{\partial f_s}{\partial p_i} < 0$ then $p_i^{\min,f_s} = \overline{p}_i$, $p_i^{\max,f_s} = \underline{p}_i$.

$$\underline{f}_s = f_s\left(\varepsilon_{cc}^{\min,f_s}, x^{\min,f_s}, p_1^{\min,f_s}, ..., p_m^{\min,f_s}\right), \tag{71}$$

$$\overline{f}_s = f_s\left(\varepsilon_{cc}^{\max,f_s}, x^{\max,f_s}, p_1^{\max,f_s}, ..., p_m^{\max,f_s}\right). \tag{72}$$

Sensitivity at the mid point is computed as

$$\frac{\partial}{\partial M} f_s = \frac{\partial f_s}{\partial \varepsilon_{cc}}\frac{\partial \varepsilon_{cc}}{\partial M} + \frac{\partial f_s}{\partial x}\frac{\partial x}{\partial M} + \frac{\partial f_s}{\partial M} \tag{73}$$

It is possible to use methods which are based on higher order derivatives in the same way as in the section 4.4 or 4.5.

5 Interval global optimization

In presented problem the number of interval parameters is relatively low, because of that to it is possible to apply interval global optimization method [4, 17]. The optimization problems are the following

$$\begin{cases} \min\ x \\ M_R - N_c \cdot (\overline{y} + d - x) = 0 \\ N_s - N_c = 0 \\ p_i \in [\underline{p}_i, \overline{p}_i] \end{cases}, \quad \begin{cases} \max\ x \\ M_R - N_c \cdot (\overline{y} + d - x) = 0 \\ N_s - N_c = 0 \\ p_i \in [\underline{p}_i, \overline{p}_i] \end{cases} \tag{74}$$

$$\begin{cases} \min\ \varepsilon_{cc} \\ M_R - N_c \cdot (\overline{y} + d - x) = 0 \\ N_s - N_c = 0 \\ p_i \in [\underline{p}_i, \overline{p}_i] \end{cases}, \quad \begin{cases} \max\ \varepsilon_{cc} \\ M_R - N_c \cdot (\overline{y} + d - x) = 0 \\ N_s - N_c = 0 \\ p_i \in [\underline{p}_i, \overline{p}_i] \end{cases} \tag{75}$$

It is possible to take into account also more complicated optimization problems

$$\begin{cases} \min\ f_s \\ M_R - N_c \cdot (\overline{y} + d - x) = 0 \\ N_s - N_c = 0 \\ p_i \in [\underline{p}_i, \overline{p}_i] \end{cases}, \quad \begin{cases} \max\ f_s \\ M_R - N_c \cdot (\overline{y} + d - x) = 0 \\ N_s - N_c = 0 \\ p_i \in [\underline{p}_i, \overline{p}_i] \end{cases} \tag{76}$$

$$\begin{cases} \min\ f_{cc} \\ M_R - N_c \cdot (\overline{y} + d - x) = 0 \\ N_s - N_c = 0 \\ p_i \in [\underline{p}_i, \overline{p}_i] \end{cases}, \quad \begin{cases} \max\ f_{cc} \\ M_R - N_c \cdot (\overline{y} + d - x) = 0 \\ N_s - N_c = 0 \\ p_i \in [\underline{p}_i, \overline{p}_i] \end{cases} \tag{77}$$

etc.

In order to find the solution first it is necessary to find some initial space where we will be looking for the solution $\mathbf{p} = [\underline{p}_1, \overline{p}_1] \times [\underline{p}_2, \overline{p}_2] \times .... \times [\underline{p}_m, \overline{p}_m]$. The algorithm of calculation is the following [4, 17].

Algorithm-3

1. Split the initial box $\mathbf{p}$ into a list of smaller boxes $L = \{\mathbf{p}_1, \mathbf{p}_2, ..., \mathbf{p}_k\}$.
2. From the list L we can reject all boxes $\mathbf{p}_i$ for which $0 \notin F_1(\mathbf{p}_i)$ or $0 \notin F_2(\mathbf{p}_i)$.
3. Calculate
$$f_{\min} = \min\left\{f\left(mid(\mathbf{p}_1)\right), ..., f\left(mid(\mathbf{p}_k)\right)\right\} \tag{78}$$
4. Remove all boxes $\mathbf{p}_i$ which satisfy the following condition
$$f_{\min} < \underline{f}(\mathbf{p}_i) \tag{79}$$
5. For all remaining boxes calculate
$$\underline{f} = \min\left\{\underline{f}(\mathbf{p}_1), ..., \underline{f}(\mathbf{p}_k)\right\} \tag{80}$$
$$\overline{f} = \min\left\{\overline{f}(\mathbf{p}_1), ..., \overline{f}(\mathbf{p}_k)\right\} \tag{81}$$
If $\overline{f} - \underline{f} < \varepsilon$ then stop calculation. Minimum is equal to the $\underline{f}$.
6. Split all remaining boxes from the list L and go to the step 2.

In order to speed up calculations it is possible to apply several procedures which are described in the books [4, 17].

6 Combinatorial solution

Combinatorial solution is obtained by considering the upper and lower bounds of the external interval moment and computing the corresponding deterministic values of ε_{cc}, x, $\overline{y}$, N_c and M_R are determined. The lower and upper values taken by these quantities are utilized to obtain the corresponding interval values of x,$\overline{y}$,z,N_c and M_R.

6.1 Example problem

A singly reinforced beam with rectangular cross section is taken up to illustrate the validity of the above methods. The beam has the dimensions b= 300 mm and D = 550 mm and effective depth d = 500 mm. The beam is reinforced with 6 numbers of Tor50 bars of 25 mm diameter ($A_S = 6 \times 491\ mm^2$). The interval bending moment acting on the beam is $M \in [96, 104]\ \ kNm$. Allowable compressive stress in concrete is $f_{co} = 13.4\ N/mm^2$and allowable strain in concrete$\varepsilon_{co} = 0.002$. Young's modulus of steel $E_S = 2.0 \times 10^5\ N/mm^2$.The stress-strain curve for concrete as detailed IS 456-1978 is adopted (Fig.1).

6.2 Results and discussion

Using equation (17) $\frac{\partial \varepsilon_{cc}}{\partial M}$ and $\frac{\partial x}{\partial M}$are computed as $\frac{\partial \varepsilon_{cc}}{\partial M} = 5.296 \times 10^{-12} \frac{1}{N \cdot mm}$and $\frac{\partial x}{\partial M} = 8.181 \times 10^{-8} \frac{1}{N}$. Similarly the values of $\frac{\partial}{\partial M} f_{cc}$ and $\frac{\partial}{\partial M} f_s$ are computed using equation (21) and equation (25) as $\frac{\partial}{\partial M} f_s = 8.429 \times 10^{-7} \frac{1}{mm^3}and\frac{\partial}{\partial M} f_{cc} = 5.354 \times 10^{-8} \frac{1}{mm^3}$. For all parameters, it is observed that the sensitivities at the endpoints have the same sign as in the midpoint, thus establishing the reliability of the solution.

Interval values of neutral axis depth x, strain ε_{cc} and stress f_{cc}in extreme compression fiber of concrete and stress in steel$f_s$computed for an external interval moment M= [96,104] kNm using search-based algorithm (SBA) and sensitivity analysis (SA) approach. Table.1a and Table 1b presents the results obtained using these two approaches along with combinatorial solution. Relative difference is computed for results obtained using SBA and SA with results obtained using combinatorial approach. It is observed that the relative difference is very small. Thus the methods agree very well with the combinatorial solution. Table 2a and Table 2b shows the corresponding results for moment of M= [60, 140] kNm corresponding to a large variation (±40percent) about the mean. It is observed that relative difference of the solution obtained using the SBA and in the range of 0.016 percent and 4.020 percent. Thus it is observed that the two approaches give reasonable bounds on the interval solution even in the presence of large uncertainty.

The interval values of bending moment at various levels of uncertainty (membership value) are shown in Fig.2. For instance, an interval bending moment of [96,104] kNm corresponds to a membership value of 0.6. A membership value of 1.0 corresponds to a point interval bending moment of [100,100] kNm. Interval values of bending moment can be extracted at any desired level of uncertainty for use in the stress analysis. The corresponding interval values of neutral axis depth, strain and stress in concrete and stress in steel reinforcement are computed at various levels of uncertainty and membership functions are plotted.

Fig. 3 shows the plots of membership function for the depth of neutral axis obtained using the approaches. The membership function for the extreme fiber stress in concrete is presented in Fig. 4. Fig. 5 depicts the membership function for the stress in steel reinforcement. Membership function for the strain in extreme fiber of concrete is shown in Fig.6. It is observed that all these membership functions are triangular with linear variation of the response about the corresponding mean value. The plots of membership functions obtained using combinatorial approach and sensitivity analysis coincide and these plots agree well with the membership functions plotted using search-based approach. Percentage variations of interval stresses in concrete (f_{cc}) and steel (f_s) and external interval bending moment M are computed about their respective mean values. Fig. 7 shows the plot of percentage variation of $\mathrm{f_S}$ and $\mathrm{f_{CC}}$as a function of the percentage variation of M. It is observed from Fig. 7 that $\mathrm{f_S}$is more sensitive to variation in bending moment in comparison to f_{CC}.

7 Conclusions

In the present paper, analysis of stresses in the cross section of a singly reinforced beam subjected to an interval external bending moment is handled by three approaches viz. a search based algorithm and sensitivity analysis and combinatorial approach. It is observed that the results obtained are in excellent agreement. These approaches allow the designer to have a detailed knowledge about the effect of uncertainty on the stress distribution of the beam. The membership functions for neutral axis depth and stresses in concrete and steel are plotted and are found to be triangular. It is observed that stress in steel is more sensitive to the given variation of bending moment when compared to the corresponding stress in concrete.

Interval stress and strain can be also calculated using sensitivity analysis. Because the sign of the derivatives in the mid point and in the endpoints is the same then the solution should be exact. More accurate monotonicity test is based on second and higher order derivatives [14]. Results with guaranteed accuracy can be calculated using interval global optimization [4].

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Table. 1a Comparison of results obtained using the three approaches for M = [96,104] kNm

	$\varepsilon_{cc}\times 10^{-4}$		f_{cc}(N/mm^2)	
	Lower	Upper	Lower	Upper
Mid-point Solution	4.9102		5.772	
Combinatorial	4.699	5.123	5.557	5.985
Search based approach	4.704	5.117	5.562	5.980
% difference	0.106	0.117	0.090	0.084
Sensitivity Analysis	4.69909	5.12276	5.55705	5.98537
% difference	0.002	0.005	0.001	0.011

Table. 1b Comparison of results obtained using the three approaches for M = [96,104]kNm				
	x(mm)		f_s(N/mm^2)	
	Lower	Upper	Lower	Upper
Mid-point Solution	270.617		83.238	
Combinatorial	270.291	270.945	79.870	86.612
Search based approach	270.299	270.937	79.543	86.972
% difference	0.003	0.003	0.409	0.416
Sensitivity Analysis	270.291	270.946	79.8712	86.6146
% difference	0.000	0.000	0.001	0.005

Table. 2a Comparison of results obtained using the two approaches for M = [60,140]kNm				
	$\varepsilon_{cc}\times10^{-4}$		f_{cc}(N/mm^2)	
	Lower	Upper	Lower	Upper
Mid-point Solution				
Combinatorial	2.859	7.107	3.558	7.831
Search based approach	2.888	7.029	3.592	7.763
% difference	1.014	1.098	0.956	0.868

Table. 2b Comparison of results obtained using the two approaches for M = [60,140]kNm				
	x(mm)		f_s(N/mm^2)	
	Lower	Upper	Lower	Upper
Mid-point Solution				
Combinatorial	267.510	274.084	49.703	117.160
Search based approach	267.553	273.958	47.663	122.127
% difference	0.016	0.046	4.104	4.240

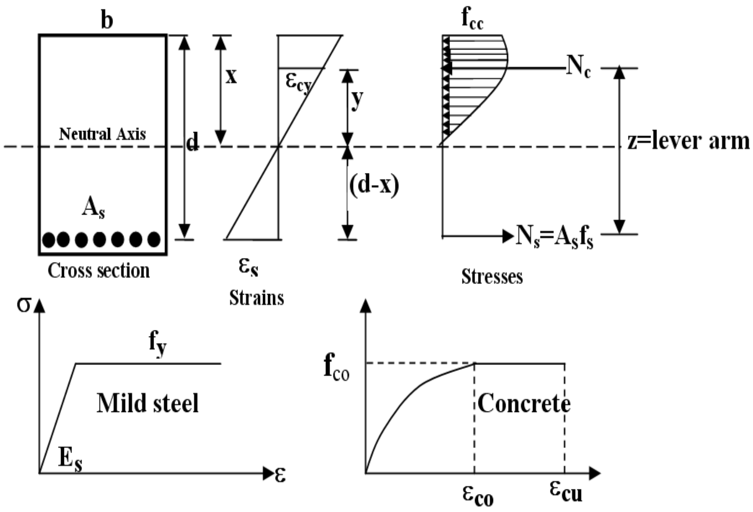

Figure 1: Stress-strain curves

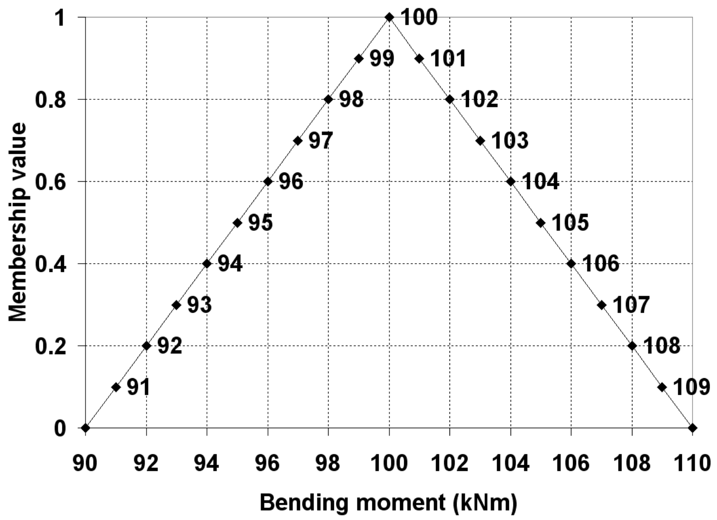

Figure 2: Bending moment

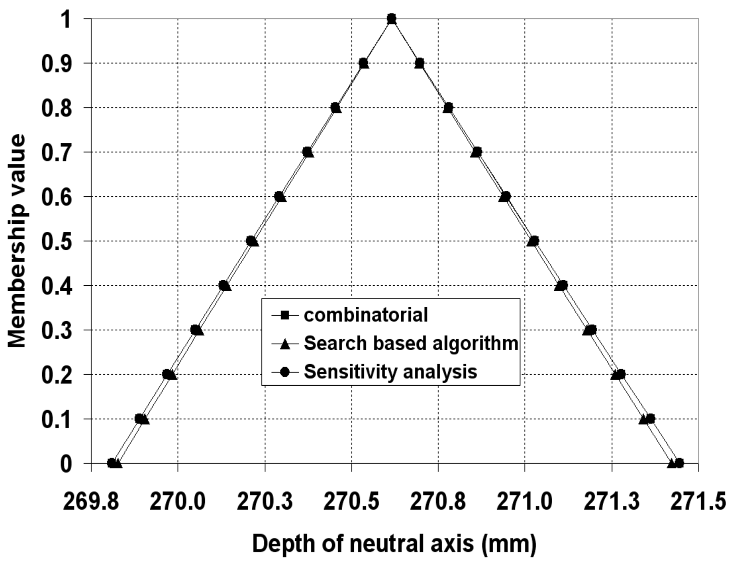

Figure 3: Membership function for depth of neutral axis

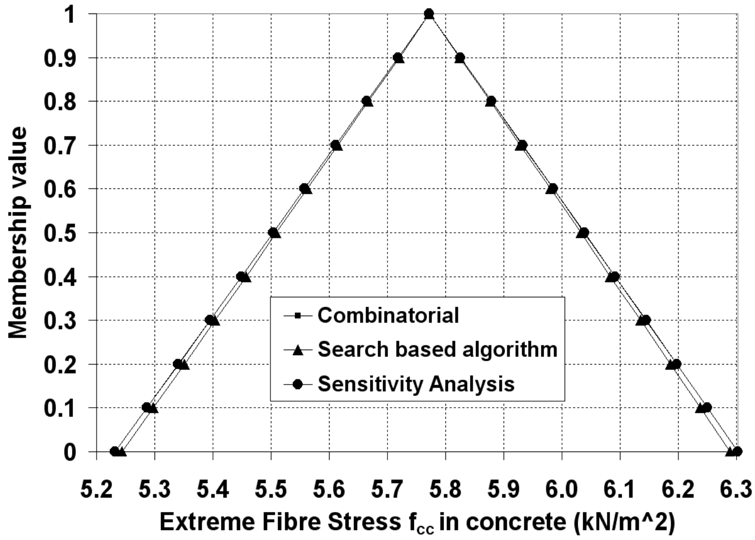

Figure 4: Membership function for stress in concrete

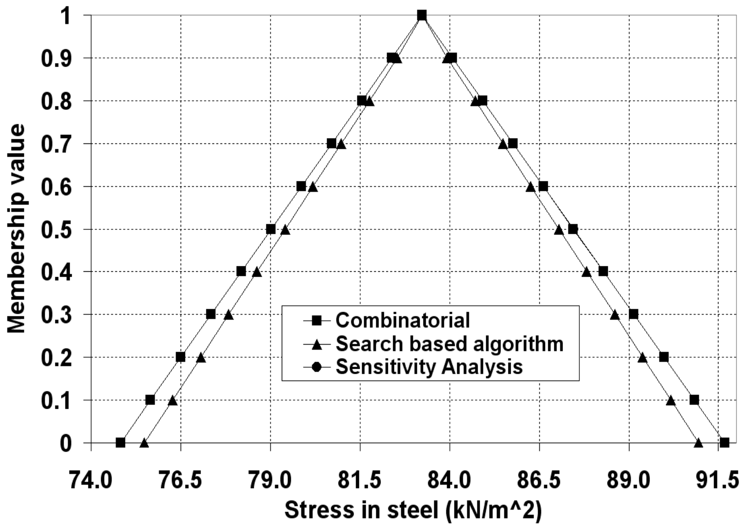

Figure 5: Membership function for stress in steel

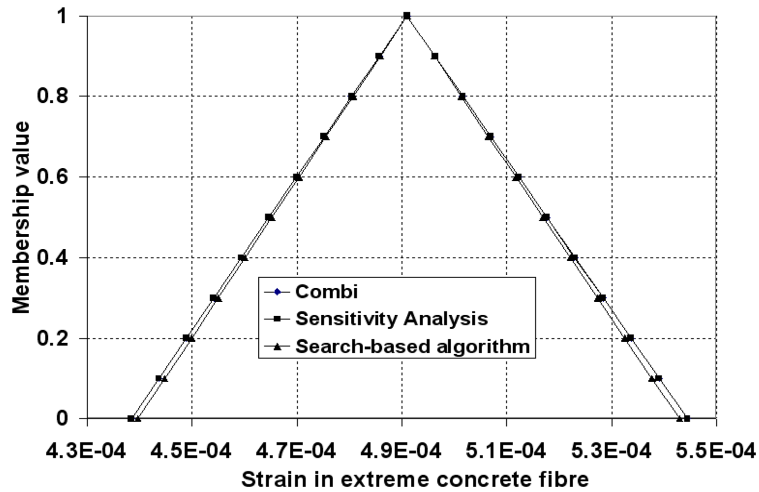

Figure 6: Membership function for strain in extreme concrete fibre

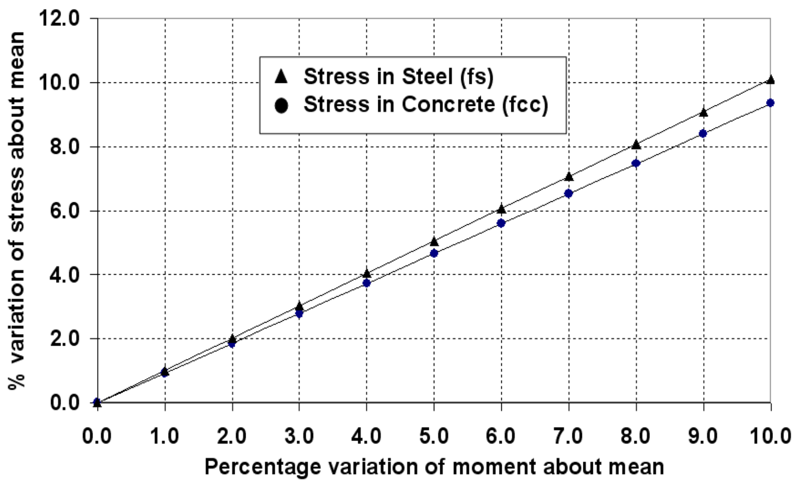

Figure 7: Sensitivity of stresses in concrete and steel