Department of Mathematical Sciences
Colloquium

Piotr Wojciechowski
Department of Mathematical Sciences, UTEP

Multiplicative Bases in Matrix Algebras

In a finite-dimensional algebra over a field $\mathbf{F}$, a basis $\mathfrak{B}$ is called *a multiplicative basis* provided that $\mathfrak{B} \cup \{0\}$ forms a semigroup. Although these bases (endowed with some additional algebraic properties) play an important role in the representation theory, their comprehensive classification for matrix algebras seems to be unknown. We will consider multiplicative bases of $\mathbf{F}_n$, the full algebra of $n \times n$ matrices over a subfield $\mathbf{F}$ of the real numbers.

As a result of the proof of the so-called Weinberg conjecture, we classified all lattice orders on $\mathbf{F}_n$. Every such lattice order corresponds to a nonsingular $n \times n$ matrix with nonnegative entries. It turns out that if the entries are either 0 or 1, the basic matrices resulting in the definition of the lattice order form a multiplicative basis, and conversely, every multiplicative basis corresponds to a nonsingular zero-one matrix. After identification of the isomorphic semigroups and also identification of the matrices that have just permuted rows and columns, the above correspondence is one-to-one. The number of zero-one nonsingular matrices, although lacking a formula so far, is known for a few small n values. This, together with the conjugacy class method from group theory allowed us to calculate the number of nonequivalent multiplicative bases up to dimension 5: 1, 2, 8, 61, 1153. We will also discuss extensions of independent multiplicative systems to multiplicative bases. This is a joint work with Carlos de la Mora.

Friday, October 21, 2005 at 3 pm. in Bell Hall 143
The University of Texas at El Paso

Refreshments will be served in front of the colloquium room, 15 minutes before the start of the colloquium.

For further information, please contact Dr. Pavel Šolín, Bell Hall 220. Phone: (915) 747-6770, email: `solin@utep.edu`.