Department of Mathematical Sciences
Colloquium

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POLYNOMIAL (BERWALD-MOORE) FINSLER METRICS AND RELATED PARTIAL ORDERS BEYOND SPACE-TIME: TOWARDS APPLICATIONS TO LOGIC AND DECISION MAKING

One of the main objectives of science and engineering is to help people select decisions which are the most beneficial to them. To make these decisions,
* we must know people's preferences,
* we must have the information about different events -- possible consequences of different decisions, and
* since information is never absolutely accurate and precise, we must also have information about the degree of certainty.

All these types of information naturally lead to partial orders:
* for preferences, $a<b$ means that b is preferable to a; this relation is used in decision theory
* for events, $a<b$ means that a can influence b; this causality relation is used in space-time physics;
* for uncertain statements, $a<b$ means that a is less certain than b; this relation is used in logics describing uncertainty such as fuzzy logic.

While an order may be a natural way of describing a relation, orders are difficult to process, since most data processing algorithms process numbers. Because of this, in all three application areas, numerical characteristics have appeared that describe the corresponding orders:
* in decision making, utility describes preferences: $a<b$ iff $u(a)<u(b)$;
* in space-time physics, metric (and time coordinates) describes causality relation;
* in logic, numbers from the interval [0,1] are used to describe degrees of certainty.

Need to combine numerical characteristics, and the emergence of polynomial aggregation formulas.
* In decision making, we need to combine utilities u_i of different participants; J. Nash showed that reasonable conditions lead to the product.
* In space-time geometry, we need to combine coordinates x_i into a metric; reasonable conditions lead to polynomial metrics such as Minkowski and Berwald-Moore's
* In fuzzy logic, we must combine degrees of certainty into a degree for $A\&B$; reasonable conditions lead to polynomial functions like product.

The fact that similar polynomials appear in different application areas indicates that there is a common reason behind them. In this talk, we provide such a general justification.
We want to find a finite-parametric class F of smooth functions $f(x_1, \dots, x_n)$ approximating the actual complex aggregation. In all three areas, the numerical values x_i are defined modulo a linear transformation. It is therefore reasonable to require that the finite-dimensional linear space F is invariant w.r.t. arbitrary linear transformations. Under this requirement, we prove that all elements of F are polynomials.
We also prove that the polynomials also appear as an optimal class F under reasonable optimization conditions.

Friday, October 9, 2009 at 3pm in Bell Hall 143
The University of Texas at El Paso

Refreshments will be served in front of the colloquium room,
15 minutes before the start of the colloquium.