Department of Mathematical Sciences Colloquium

An Overview of the Asymptotics for Solutions to a Class of Evolution Equations and Difference Equations

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Abstract

We present our recent results on the asymptotic behavior of bounded solutions to the following class of second order nonhomogeneous evolution equations

$$\begin{cases} u''(t) - cu'(t) \in Au(t) + f(t) \text{ a.e. } t \in (0, +\infty) \\ u(0) = u_0, \ \sup_{t \ge 0} |u(t)| < +\infty \end{cases}$$

where A is a monotone operator in a real Hilbert space H, $c \ge 0$, and $f : \mathbb{R}^+ \to H$ is a given function, as well as its discrete analogue corresponding to the following class of second order difference equations

$$\begin{cases} u_{n+1} - (1+\theta_n)u_n + \theta_n u_{n-1} \in c_n A u_n + f_n & n \ge 1\\ u_0 = a \in H, & \sup_{n \ge 0} |u_n| < +\infty \end{cases}$$

where $\{c_n\}$ and $\{\theta_n\}$ are positive real sequences and $\{f_n\}$ is a sequence in H. With suitable conditions on A and the sequences $\{c_n\}$, $\{\theta_n\}$ and $\{f_n\}$, we prove ergodic theorems, as well as weak and strong convergence theorems for solutions to these equations, converging to an element of $A^{-1}(0)$, which is also the asymptotic center of the sequence $\{u_n\}$, implying therefore in particular that the existence of a solution $\{u_n\}$ implies that $A^{-1}(0) \neq \emptyset$. Our results extend and give simpler proofs to previous results by several authors who studied special cases of similar problems by assuming that $A^{-1}(0) \neq \phi$, and have many applications in approximation and optimization theory.

Friday, October 16, 2009 at 3 pm in Bell Hall 143 The University of Texas at El Paso

Refreshments will be served in front of the colloquium room, 15 minutes before the start of the colloquium.