Colloquium

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GEOMETRY AND THE IMAGINATION

Ramifications Of The Inscribed Circle Theorem For Pythagorean Triplets

The undergraduate level courses in Elementary Number Theory include a discussion of Pythagorean triplets. The classification of primitive Pythagorean triplets and hence all Pythagorean triplets of natural numbers can be found in the book, David M. Burton, Elementary Number Theory, Fifth Edition, McGraw Hill Company in pages 234-239. The Inscribed Circle Theorem given in page 239, asserts the positive integral radius for the inscribed circle of an associated right triangle that can be linked with a given Pythagorean triplet of natural numbers. Most of the time when the side lengths of a Euclidean triangle are taken to be valid positive integers, the inscribed circle of that triangle need not have a positive integral radius. It is more conspicuous in the case of an equilateral triangle that has a positive integer as the side length. In my talk, which is based on my published paper, Circles of Pythagorean Triplets of Natural Numbers I have shown the following: (1) For any arbitrary Pythagorean triplet(primitive or not) there are at least nine circles of positive integral radii that include the well known four pack family of circles consisting of the inscribed circle, and the three escribed circles of the associated right triangle. (2) When the three components of the chosen Pythagorean triplet are all divisible by four, there are at least twelve circles of positive integral radii that include the six pack family of circles consisting of inscribed circle, the three escribed circles, the circumcircle and the nine-point circle of the right triangle . The remaining six circles are less well known. Most of the time concentric circles around points where already circles of positive integral radii exist are excluded by self imposed restrictions. The ramifications cover a very interesting concrete example of a Pythagorean triplet for which there are as many as 16 circles of positive integral radii, provided a few justifiable concentric circles are included. Of course this would include the six pack family of circles mentioned earlier. A discussion of the celebrated Feuerbachs Theorem is made possible on account of this example. This example led to further explorations, resulting in characterizations of right triangles and equilateral triangles in terms of the nine-point circles. Other representations of Pythagorean triplets based on ellipses will also be mentioned in the talk. It is further believed that this talk, mostly in the form of survey talk, would cover some Advanced Trigonometry related Geometry that are not usually taught in college courses any more. It will be of some use to students in M. A. degree in teaching (MAT program). The talk is very elementary in nature and is expected to be a pleasant excursion in the area of Geometric Number Theory.