Department of Mathematical Sciences
The University of Texas at El Paso

Colloquium

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Friday, October 18, 2013 at 3 pm in Bell Hall 143

Turbulent Particle Pair Diffusion:locality versus non-locality

Richardson's 1926 theory for particle pair diffusion in a field of homogeneous turbulence is based upon the hypothesis of locality in which the pair diffusion process is governed by correlations that extend over scales of the same order as the pair separation $\Delta = |\Delta(t)|$ itself and by the rate of turbulent kinetic energy dissipation per unit mass ε.

Here, we show that the pair diffusion in turbulence with generalised power-law energy spectra, $E(k) \sim k^{-p}$ for $1 < p < 3$, is in fact governed by both *local and non-local correlations*, leading to relative diffusivities of the form $D_p \approx D_p^l + D_p^{nl}$ where the local component scales like $D_p^l \sim \varepsilon^{1/3} \lambda^{(5/3-p)/2} \sigma_\Delta^{(1+p)/2}$; and the non-local component scales like $D_p^{nl} \sim \varepsilon^{1/3} \lambda^{-2/3} \sigma_\Delta^2$. $\sigma_\Delta = \sqrt{\langle \Delta^2 \rangle}$, and λ is the Taylor microscale which bears an influence even in the 'local' term, so locality in the sense of Richardson is never observable. The angled brackets denote ensemble average.

In the limit $p \to 1$, we obtain $D_p / D_p^{nl} \gg 1$ yielding $D_p \to D_p^l \sim \varepsilon^{1/3} \lambda^{2/3} \sigma_\Delta$. In the limit $p \to 3$, we obtain $D_p / D_p^{nl} \ll 1$ yielding $D_p \to D_p^{nl} \sim \varepsilon^{1/3} \lambda^{-2/3} {\sigma_\Delta}^2$. But due to the inherent uncertainties in defining the 'size' of a turbulent eddy, the relative balance of the two components cannot be estimated in the general case, $1 < p < 3$; nevertheless the relative diffusivity must be a smooth transition between the two asymptotic cases manifesting as a power law $D_p \sim {\sigma_\Delta}^{\gamma_p}$ with γ_p such that as $p \to 1$ then $\gamma_p \to 1$, and as $p \to 3$ then $\gamma_p \to 2$; in all other cases γ_p must be determined from experiment or from simulation, although we expect it to be greater than from the locality scaling alone, $\gamma_p > (1+p)/2$.
Numerical results using Kinematic Simulation (Kraichnan 1970, Fung et al 1992) confirms these predictions and strongly supports the fundamental idea behind the theory that local and non-local correlations govern the turbulent pair diffusion process in the inertial range.
Objections raised by Thomson and Devenish (2005), and Nicolleau and Nowakowski (2011) that KS is unsuitable for diffusion studies are found to be flawed. Rather, the KS method yields accurate and physically reliable results, provided that the results are re-plotted against an adjusted time origin as proposed by Batchelor (1952) and Fung et al (1992).

For further information, please contact Dr. Emil Schwab, eschwab@utep.edu. Refreshments will be served in front of the colloquium room, 15 minutes before the start of the colloquium.