Department of Mathematical Sciences
The University of Texas at El Paso

Colloquium

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Friday, September 04, 2015 at 3pm in Bell Hall 143

Some Problems and Progress in Affine Algebraic Geometry

Affine algebraic geometry applies algebraic methods to study geometric properties of algebraic sets, i.e. sets which can be de.ned as the set of common zeros of a collection of polynomials in affine space k^n where k is some field (for this presentation k will almost always be the .eld of complex numbers). The subject is noteworthy for its abundance of natural and easily stated problems which remain unsolved. These include:

- The Jacobian problem: A morphism from an algebraic set $X \subset k^n$ to an algebraic set $Y \subset k^m$ is given by an m–tuple of polynomials in n variables well defined on X and with image in Y. An isomorphism is a morphism with a 2-sided inverse (also a morphism). Let $n \geq 2$ and $F := (f_1, \ldots, f_n) : \mathbb{C}^n \to \mathbb{C}^n$ a morphism with Jacobian matrix $JF = (\partial f_i / \partial x_j)$. Is F an isomorphism provided $\det(JF)$ is a nonzero element of $\mathbb{C}$?
- The Dolgachev–Weisfeiler conjecture: If $k \geq 2$ and $G := (g_1, \ldots, g_n) : \mathbb{C}^{n+m} \to \mathbb{C}^n$ is a morphism all of whose fibers are isomorphic to m-dimensional affine spaces, is there a change of variables in which F is the projection onto the first n coordinates?
- The cancellation problem: If an algebraic set X satisfies $X \times \mathbb{C}^1 \cong \mathbb{C}^4$, is $X \cong \mathbb{C}^3$?
- The embedding problem: If $X \subset \mathbb{C}^3$ is isomorphic to $\mathbb{C}^1$, is there an isomorphism $F := (f_1, f_2, f_3) : \mathbb{C}^3 \to \mathbb{C}^3$ for which $F(X)$ is the coordinate line $x_2 = x_3 = 0$?

After some introductory information on algebraic sets, progress on these problems will be discussed (as many and in as much detail as time permits).

For further information, please contact Dr. Emil Schwab, eschwab@utep.edu
Refreshments will be served in front of the colloquium room,
15 minutes before the start of the colloquium.