Cost, Revenue & Profit Examples

1) A soft-drink manufacturer can produce 1000 cases of soda in a week at a total cost of $6000, and 1500 cases of soda at a total cost of $8500. Find the manufacturer’s weekly fixed costs and marginal cost per case of soda.

Solution: We would like to find a function $C(x) = mx + b$ that describes this situation. Recall that the marginal cost is $m$, and in this case it represents the cost to produce one case of soda. The fixed costs are represented by $b$. To answer the questions, we must find both $m$ and $b$. Since this is a linear function, we can find $m$ using the formula for finding the slope of a line between two points, where the points here are $(x_1, C_1)$ and $(x_2, C_2)$. The slope formula is

$$m = \frac{C_2 - C_1}{x_2 - x_1} = \frac{8500 - 6000}{1500 - 1000} = 5$$

So the marginal cost is $5. This means it costs $5 per case to manufacture the soft-drinks. Now that we have $m$, we can find $b$. We know that $C(x) = 5x + b$, so we can use what we were given to find $b$. You can choose either ordered pair to do this, either $(1000, 6000)$ or $(1500, 8500)$. We plug in the $x$ and $C$ values we know into our equation, and solve for $b$:

$$6000 = 5(1000) + b, \text{ or } 6000 = 5000 + b$$

So $b = 1000$. Thus the fixed costs are $1,000. This would represent things like rent, electricity and other bills, etc.

2) Your college newspaper, *The Collegiate Investigator*, has fixed production costs of $70 per edition, and marginal printing and distribution costs of 40¢/copy. *The Collegiate Investigator* sells for 50¢/copy.

a) Write down the associated cost, revenue, and profit functions.

b) What profit (or loss) results from the sale of 500 copies of *The Collegiate Investigator*?

c) How many copies should be sold in order to break even?

Solutions:

a) We are told that the marginal cost is $0.40 per copy, and that the fixed costs are $70. This means that our cost function is

$$C(x) = 0.40x + 70$$

Since they are selling the newspaper for $0.50 per copy, the revenue function is

$$R(x) = 0.50x$$

Since profit is defined to be revenue minus cost, the profit function is

$$P(x) = 0.50x - (0.40x + 70) = 0.10x - 70$$
b) We can find the profit that results from selling 500 copies by finding $P(500)$, that is, plugging 500 into the profit function. We get

$$P(500) = 0.10(500) - 70 = -20$$

This means that if they sell 500 newspapers, it will result in a loss of $20.

c) To find the break even quantity, we can either set revenue equal to cost and solve for $x$, or we can set profit equal to zero and solve for $x$. If we set profit equal to zero, we get

$$0.10x - 70 = 0$$

Solving this for $x$ gives $x = 700$, so they must sell 700 newspapers in order to break even.

3) The cost of renting tuxes for the Choral Society's formal is $20 down, plus $86 per tux. Express the cost $C$ as a function of $x$, the number of tuxedos rented.

Solution: Since it costs $20 regardless of how many tuxes you rent, this is the fixed cost. Since it's $86 per tux, this is the marginal cost. The cost function is thus $C(x) = 86x + 20$.

Use your function to answer the following questions.

a) What is the cost of renting 5 tuxes?

Solution: Plug 5 into the cost function to find the cost to rent 5 tuxes:

$$C(5) = 86(5) + 20 = 450$$

So it costs $450 to rent 5 tuxes.

b) What is the cost of the 5th tux?

Solution: The cost of just the 5th tux (not the cost to rent 5 tuxes!) is $86 (which is the marginal cost, the cost per tux).

c) What is the cost of the 3098th tux?

Solution: The cost of the 3098th tux is $86. (Note that every tux, individually, costs $86 to rent).

d) What is the variable cost?

Solution: The variable cost is $86x$.

e) What is the marginal cost?

Solution: The marginal cost is $86$. 
f) Graph the cost $C$ as a function of $x$.

Solution: The graph looks like this: Note that the $C$-intercept is at 20.

![Graph of Cost as a Function of x](image)

4) Gymnast Clothing manufactures expensive hockey jerseys for sale to college bookstores in runs of up to 150. Its cost (in dollars) for a run of $x$ hockey jerseys is

$$C(x) = 1500 + 10x + 0.2x^2, \quad (0 \leq x \leq 150)$$

a) Gymnast Clothing sells the jerseys at $90 each. Find the revenue function.

Solution: Since the manufacturer sells the jerseys for $90 each, the revenue function is

$$R(x) = 90x$$

b) Find the profit function.

Solution: Profit is defined to be revenue minus cost, so the profit function is

$$P(x) = R(x) - C(x) = 90x - (1500 + 10x + 0.2x^2) = -1500 + 80x - 0.2x^2$$

c) How many should Gymnast Clothing manufacture to make a profit? (Round your answer up to the nearest whole number.)

Solution: In order to make a profit, $P(x)$ must be greater than zero. So, to find how many jerseys we need to make in order to make a profit, we should find the break-even point. We can do this by setting profit equal to zero and solving for $x$. This will require the quadratic formula:

$$-0.2x^2 + 80x - 1500 = 0$$

$$x = \frac{-80 \pm \sqrt{80^2 - 4(-0.2)(-1500)}}{2(-0.2)}$$

This simplifies to $x = 19.7$ or $x = 380.2$, but since the problem specifies that the domain is between $x = 0$ and $x = 150$, we can reject the larger answer. Rounding the other answer up, we get $x = 20$. So, if we make 20 jerseys, we will make a profit.