Pack-Em-In Real Estate is building a new housing development. The more houses it builds, the less people will be willing to pay, due to the crowding and smaller lot sizes. In fact, if it builds 40 houses in this particular development, it can sell them for $320,000 each, but if it builds 50 houses, it will only be able to get $300,000 each. Obtain a linear demand equation. (Let p be the price of a house and q the number of houses.)

Solution: We need to find a linear demand function with q as the independent variable. This means it needs to have the form \( p(q) = mq + b \). Note that this is not the usual way that a demand function is written in this course—it’s usually \( q(p) = mp + b \) (with p as the independent variable). For this problem, we can find \( m \) as follows:

\[
m = \frac{p_2 - p_1}{q_2 - q_1} = \frac{300,000 - 320,000}{50 - 40} = \frac{-20,000}{10} = -2000.\]

The equation so far is \( p(q) = -2000q + b \). We can find \( b \) by plugging in one of the price/quantity pairs given in the problem: \( 300,000 = -2000(50) + b \). Solve this for \( b \) to get \( b = 400,000 \), so \( p(q) = -2000q + 400,000 \).

Determine how many houses Pack-Em-In should build to get the largest revenue.

Solution: Recall that Revenue = (price) \times (quantity), so

\[
R(q) = pq = (-2000q + 400,000)q = -2000q^2 + 400,000q.
\]

This is a quadratic function with negative leading coefficient. Therefore it opens down, and its vertex is a maximum. So, to find the number of houses they should build to get the largest revenue, we must find the q-coordinate of the vertex. To do this, we must use the vertex formula (found on p. 621 of the textbook):

\[
q_{\text{max}} = \frac{-400,000}{2(-2000)} = 100.
\]

So, they must build 100 houses to maximize revenue.

What is the largest possible revenue?

Solution: To find the maximum possible revenue, we need to plug in our answer for the previous part into the revenue function:

\[
R_{\text{max}} = R(100) = -2000(100)^2 + 400,000(100) = 20,000,000.
\]

The maximum possible revenue is $20,000,000.