The Eigenvalue Problem
For an \( n \times n \) matrix \( A \), find all scalars \( \lambda \) such that the equation
\[
Ax = \lambda x
\]
has a nonzero solution, \( x \). Such a scalar \( \lambda \) is called an eigenvalue of \( A \), and any nonzero \( n \times 1 \) vector \( x \) satisfying the equation above is called an eigenvector corresponding to \( \lambda \).

**Problem 1.** Find the eigenvalues and the eigenvectors for the given matrix.

a) \[ A = \begin{bmatrix} 3 & -1 \\ 5 & -3 \end{bmatrix} \]

b) \[ A = \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix} \]

**Problem 2.** Show that there is not real scalar \( \lambda \) such that \( A - \lambda I \) is singular.

a) \[ A = \begin{bmatrix} 3 & -2 \\ 5 & -3 \end{bmatrix} \]

b) \[ A = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}, \ b \neq 0 \]

**Problem 3.** Let \( A \) be a \( 2 \times 2 \) matrix. Show that \( A \) and \( A^T \) have the same set of eigenvalues.

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Homework: Read Section 4.1, do 1-17 (odd).