Section 4.3

**Theorem**
If $A$ is an $n \times n$ matrix, then
\[
\det(A) = \det(A^T).
\]

**Theorem**
Let $A$ be an $n \times n$ matrix, and let $B$ be formed by interchanging any two rows (or columns) of $A$. Then
\[
\det(B) = -\det(A).
\]

**Corollary**
If $A$ is an $n \times n$ matrix with two identical rows (columns), then $\det(A) = 0$.

**Theorem**
Suppose that $B$ is obtained from the $n \times n$ matrix $A$ by multiplying one row (or column) of $A$ by a nonzero scalar $c$ and leaving the other rows (or columns) unchanged. Then
\[
\det(B) = c\det(A).
\]

**Theorem**
Let $A$ be an $n \times n$ matrix. Suppose that $B$ is the matrix obtained from $A$ by replacing the $i$th row of $A$ by the $i$th row of $A$ plus a constant multiple of the $k$th row of $A$, $k \neq i$. Then
\[
\det(B) = \det(A).
\]

**Theorem**
An $n \times n$ matrix $A$ is singular if and only if $\det(A) = 0$.

**Problem 1.** Use only column interchanges or row interchanges to produce a triangular determinant and then find the value of the original determinant.

\[
A = \begin{vmatrix}
0 & 1 & 0 & 0 \\
0 & 2 & 0 & 3 \\
2 & 1 & 0 & 6 \\
3 & 2 & 2 & 4
\end{vmatrix}.
\]

**Problem 2.** Assume that the $3 \times 3$ matrix $A$ satisfies $\det(A) = 2$, where $A$ is given by
\[
A = \begin{bmatrix}
a & b & c \\
d & e & f \\
g & h & i
\end{bmatrix}
\]
Calculate $\det(B)$ in each case.

\[
B = \begin{bmatrix}
d & e & f \\
g & h & i \\
a & b & c
\end{bmatrix}
\quad B = \begin{bmatrix}
a & b & c \\
a + d & b + e & c + f \\
2g & 2h & 2i
\end{bmatrix}
\]

**Problem 3.** Use row operations to obtain a triangular determinant and find the value of the original Vandermonde determinant.

\[
\begin{vmatrix}
1 & a & a^2 & a^3 \\
1 & b & b^2 & b^3 \\
1 & c & c^2 & c^3 \\
1 & d & d^2 & d^3
\end{vmatrix}
\]

**Problem 4.** Let $A$ be an $n \times n$ matrix. Show that $\det(cA) = c^n \det(A)$.

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Homework: Read Section 4.3, do 5-23 (odd).