Undesirable Habits of Mind of Pre-service Teachers: Strategies for Addressing Them

Kien Lim
University of Texas at El Paso

AMS-MAA-MER Special Session
JMM 2009
Outline of Presentation

- Habits of Mind (HoM): An overview
- An undesirable HoM: Impulsive disposition
- Strategies for addressing impulsive disposition
- Some encouraging results
Habits of Mind (HoM)

- Organizing principle for mathematics curricula
- General HoM
  - They cut across every discipline
  - E.g. pattern sniffing, experimenting, thinkering, inventing, visualizing, conjecturing, guessing
- Mathematical HoM
  - “the way mathematicians think about problems”
  - E.g. talk big think small, talk small think big, use functions, use multiple points of view, mix deduction and experiment

Cuoco, Goldenberg, Mark (1996)
Ways of Thinking

- HoM are internalized *ways of thinking*.

- Mathematics consists of two complementary subsets.
  - *Ways of understanding* are the products of doing mathematics, which include axioms, definitions, theorems, proofs, problems and solutions.
  - *Ways of thinking* are conceptual tools for constructing those products.

- Harel’s Duality Principle
  “Students develop ways of thinking only through the construction of ways of understanding, and the ways of understanding they produce are determined by the ways of thinking they possess.”

  Harel (2007, 2008)
1. Beliefs

- **Mathematics** is a collection of rules and procedures.
- “Doing mathematics means following the rules laid down by the teacher, knowing mathematics means remembering and applying the correct rule when the teacher asks a question, and mathematical truth is determined when the answer is ratified by the teacher.”
  
  (Lampert, 1990, p. 31)
Habits of Mind: An Overview

Undesirable WoT

1. Beliefs
   - Mathematics is a collection of rules and procedures.

2. Proof-schemes
   - Authoritative proof scheme
   - Empirical proof scheme (Harel & Sowder, 1998)

3. Problem-solving approaches
   - “Waiting to be told what to do”
   - “Doing whatever first comes to mind ... or diving into the first approach that comes to mind”
      (Watson & Mason, 2007, p. 207)
Evidence #1

A group of 5 musicians plays a piece of music in 10 minutes. Another group of 35 musicians will play the same piece of music. How long will it take this group to play it?

\[ \frac{5}{10} = \frac{.5}{x} \]

Cross multiply to figure out since it is proportional:

\[ 5x = 350 \]

\[ x = \frac{350}{5} \]

\[ x = 70 \text{ minutes or } 1 \text{hr } 10 \text{min} \]

Only 47% recognized that the answer is 10 minutes. 42% obtained 70 minutes, and 11% found other numbers.
Evidence #2

Gina is traveling home from her friend’s house. The graph represents a portion of Gina’s journey. What is Gina’s speed at the 20th minute?

(a) Approximately 3000 meters
(b) Approximately 50 meters/min
(c) Approximately 80 meters/min
(d) Approximately 150 meters/min

Answer: 28% 52%
Evidence #2

Gina is traveling home from her friend’s house. The graph represents a portion of Gina’s journey. What is Gina’s speed at the 20th minute?

Answer: (d) Approximately 150 meters/min

28% 52%
Evidence #2

Gina is traveling home from her friend’s house. The graph represents a portion of Gina’s journey. What is Gina’s speed at the 20th minute?

Distance from home (meters)

Time (min)

(a) Approximately 3000 meters
(b) Approximately 50 meters/min
(c) Approximately 80 meters/min
(d) Approximately 150 meters/min

Answer: A B C D

307 Pre-service EC-4 Teachers
An Undesirable HoM: Impulsive Disposition

Two Possible Explanations

- **Human Nature**
  
  “Our thinking is canalized with respect to the way we have learned to deal with things ... we implicitly anticipate that similar issues have similar causes, and thus similar solutions.”  
  (Reigler, 2001, p. 535)

- **School Effect (i.e. Nurture)**
  
  - Compartmentalization of school mathematics
  
  - Emphasis on procedures for solving routine problems
Practical Suggestions

- Do not teach algorithms/formulas prematurely
- Pose problems that
  - necessitate a particular algorithm/concept

A new housing subdivision offers rectangular lots of three different sizes:
- a. 75 feet by 114 feet
- b. 455 feet by 508 feet
- c. 185 feet by 245 feet

If you were to view these lots from above, which would appear most square?

(Simon & Blume, 1994)
Practical Suggestions

- Do not teach algorithms/formulas prematurely
- Pose problems that
  - necessitate a particular algorithm/concept

“Students are most likely to learn when they see a need for what we intend to teach them, where by ‘need’ is meant intellectual need, not social or economic need.”  (Harel, 1998, p. 501)
Practical Suggestions

- Do not teach algorithms/formulas prematurely
- Pose problems that
  - necessitate a particular algorithm/concept
  - intrigue students
Intriguing Students

Question: Which method should I use?
Practical Suggestions

- Do not teach algorithms/formulas prematurely
- Pose problems that
  - necessitate a particular algorithm/concept
  - intrigue students
  - require students to attend to meaning of numbers/symbols
Question: What does 39 mean? What does 0.90 mean?
Question: What does 39 mean?
Strategies for Addressing Impulsive Disposition

Attending to Meaning

Question: What does 0.90 mean?

Ratio = \frac{\text{Width}}{\text{Length}}

455 feet is 0.9 times of 508 feet

Ratio as a multiplicative comparison
Practical Suggestions

- Do not teach algorithms/formulas prematurely
- Pose problems that
  - necessitate a particular algorithm/concept
  - intrigue students
  - require students to attend to meaning of numbers/symbols
  - require students to explain and justify
Question: Why is the ratio method better?

\[
\text{Ratio} = \frac{\text{Width}}{\text{Length}}
\]

\[
\text{Difference} = \text{Length} - \text{Width}
\]

\[
\text{Area of Square} = \text{Width} \times \text{Width}
\]

\[
\text{Area of Rectangle} = \text{Length} \times \text{Width}
\]

\[
\text{Ratio} = \frac{\text{Area of Square}}{\text{Area of Rectangle}}
\]
Practical Suggestions

- Do not teach algorithms/formulas prematurely
- Pose problems that
  - necessitate a particular algorithm/concept
  - intrigue students
  - require students to attend to meaning of numbers/symbols
  - require students to explain and justify
- Include contra-problems to promote skepticism
Sharon and Terri were comparing the size of their palms. Who do you think has a larger palm?

9 students compared ratios.
3 students compared differences.
10 students compared areas.

\[
\frac{90}{84} = 1.071 \quad \frac{105}{70} = 1.5
\]

Terri has a bigger palm because the ratio of her palm's height to her width is greater than Sharon's.
Sharon and Terri were comparing the size of their palms. Who do you think has a larger palm?

9 students compared ratios.
3 students compared differences.
10 students compared areas.

Identifying numbers and selecting operations

versus

Analyzing quantities and relationships
Sharon and Terri were comparing the size of their palms. Who do you think has a larger palm?

A Student’s Written Comment:
“Dr. Lim had the great art of using awesome little tricks that would make us think [that] you [should] use ratios, for example, when in fact it was multiplication! This was a great tactic, because often I would rush right into what I had just been taught, not even looking into the problem.”
Practical Suggestions

- Do not teach algorithms/formulas prematurely
- Pose problems that
  - necessitate a particular algorithm/concept
  - intrigue students
  - require students to attend to meaning of numbers/symbols
  - require students to explain and justify
- Include contra-problems to promote skepticism
- Include superficially-similar-but-structurally-equivalent problems in tests and exams

Strategies for Addressing Impulsive Disposition
Some Encouraging Results

Fall 07 vs. Fall 08

Direct-Proportional Item
Fall 07 (47 students) Fall 08 (66 students)
The ratio of the amount of soda in the can to the amount of soda in the bottle is 4:3. There are 12 fluid ounces of soda in the can, how many fluid ounces of soda are in the bottle?

- (a) 8 fluid ounces
- (b) 9 fluid ounces
- (c) 15 fluid ounces
- (d) 16 fluid ounces
- (e) None of the above

Inverse-Proportional Item
The ratio of the volume of a small glass to the volume of a large glass is 3:5. If it takes 15 small glasses to fill the container, how many large glasses does it take to fill the container?

- (a) 9 glasses
- (b) 13 glasses
- (c) 17 glasses
- (d) 25 glasses
- (e) None of the above
Students’ Written Comments

- Do not teach algorithms/formulas prematurely

“My experience in this course was different from that in other classes because in this class ... explanation did not come until after we worked on the problem, or after we were assessed. ... It has been difficult for me to do math this new way, because I have been taught a different way of doing math for over twelve years. It would take more than just one semester of this kind of math for me to actually make it a habit.

- Include problems that require thinking in quizzes, tests and exams
Some Encouraging Results

Students’ Written Comments

“I learned to analyze the problem instead of rushing into a procedure, I used to do that.”

■ Pose problems that
  o necessitate a particular algorithm/concept
  o intrigue students
  o require students to attend to meaning of numbers/symbols
  o require students to explain and justify

“I think that this class helped me ... by thinking deeper about that problem instead of just looking at the numbers and wanting to do something with them.”
Some Encouraging Results

Students’ Written Comments

- Do not teach algorithms/formulas prematurely
- Pose problems that
  - necessitate a particular algorithm/concept
- Include contra-problems to promote skepticism
- Include problems that require thinking in quizzes, tests and exams

“In this class, the concepts remain the same, yet the problems themselves are always quite different. I can no longer rely on ‘similar problems’ in order to figure out my homework or pass [the] exams.”
Students’ Written Comments

“This class is very demanding because I have to dedicate more time to learn how to get rid of those “bad habits” that I have learned in previous classes.”

“It would take more than just one semester of this kind of math for me to actually make it a habit.”
Thank You