Math 2313, Test I

Name ___________________________

1. Find the equations (parametric or symmetric) of the line:

   a. perpendicular to the plane $2x + 3y - 2z - 8 = 0$ and through the point $(2, 2, 3)$
   answer: $x = 2 + 2t, y = 2 + 3t, z = 3 - 2t$ or $\frac{x-2}{2} = \frac{y-2}{3} = \frac{z-3}{-2}$

   b. through the points $(0, 0, 3), (2, 1, 0)$.
   answer: $(2, 1, 0)-(0, 0, 3) = \langle 2, 1, -3 \rangle > so x = 2t, y = t, z = 3-3t$
   (many other forms possible)

2. Consider the two planes $4x - 3y + 2z = 12$ and $x + 5y - z = 25$.

   a. Find a vector parallel to the line of intersection of these planes.
   answer: $(-7, 6, 23)$

   b. Find the angle between the two planes (at the intersection)
   answer: $\theta = 117.68$ (or 62.32) degrees
3. Find the point of intersection (if any) of the plane \(2x - 2y + z = 10\) and the line \(x = 1 + 4t, y = 2t, z = 3 + 6t\).
   answer: \((3, 1, 6)\)

4. If \(r(t) = < \sin(\pi t^2), \cos(\pi t^2), t^2 >\), find the velocity \(r'(t)\) and the speed, \(||r'(t)||\).
   answer: \(r'(t) = < 2t\pi \cos(\pi t^2), -2t\pi \sin(\pi t^2), 2t >\), \(||r'(t)|| = 2|t|\sqrt{\pi^2 + 1}\)

5. Find the length of the helix of problem 4, from \(t = 0\) to \(t = 2\).
   answer: \(\int_0^2 ||r'(t)||dt = \int_0^2 2|t|\sqrt{\pi^2 + 1} dt = 4\sqrt{\pi^2 + 1}\)

6. If the acceleration of an object is \(r''(t) = < t, -1 >\), find the position vector \(r(t) = < x(t), y(t) >\), if \(r(0) = < 0, 1 >\) and \(r'(0) = < 1, 0 >\).
   answer: \(r(t) = < t^3/6 + t, -t^2/2 + 1 >\)