Math 2313, Test II

Name ___________________________

1. If \( f(x, y) = \sin(xy + x^2 + y^2) \),
   a. Find the gradient of \( f \) at \((0, \sqrt{\pi})\).
      answer: \((-\sqrt{\pi}, -2\sqrt{\pi})\)

   b. Find the derivative of \( f \) at \((0, \sqrt{\pi})\) in the direction of the vector \(< 3, -4 >\).
      answer: \(\sqrt{\pi}\)

   c. What is the maximum directional derivative, at the point \((0, \sqrt{\pi})\)?
      answer: \(\sqrt{5\pi}\)

2. If \( f(x, y, z) = e^{x^2 y z^3} \) find \( f_{xy} \).

   answer: \((2xz^3 + 2x^3yz^6)e^{x^2yz^3}\)
3. A cylinder has radius \( r = 6 \) and height \( h = 10 \), and the radius and height are changing at the rate of \( \frac{dr}{dt} = 0.1, \frac{dh}{dt} = -0.2 \). How fast is the surface area changing (\( \frac{dA}{dt} \)), given that the surface area is \( A = 2\pi(r^2 + rh) \).

answer: \( 2\pi \)

4. If \( f(x, y) = x^3 - 12xy + y^3 \) find all critical points and classify each as a maximum, minimum or saddle point.

answer: \((4,4)\) is a minimum, \((0,0)\) is a saddle point

5. Find the equation of the tangent plane to the surface \( z = x^2 - y^2 \), at the point \((3,2,5)\).

answer: \( 6x - 4y - z = 5 \)