1. If \( f(x, y, z) = e^{2x+2y-z^2} \),
   a. Find the gradient of \( f \) at \((1, 1, 2)\).
      answer: \( (2, 2, -4) \)
   
   b. Find the derivative of \( f \) at \((1, 1, 2)\) in the direction of the vector
      \(<0, 3, 4>\).
      answer: \(-2\)
   
   c. In what direction is the directional derivative largest, at the point
      \((1, 1, 2)\)?
      answer: \( (2, 2, -4) \)
   
   d. Find the equation of the tangent plane to the surface \( f(x, y, z) = 1 \)
      at \((1, 1, 2)\).
      answer: \( 2x + 2y - 4z + 4 = 0 \)

2. If \( f(x, y) = e^{xy} \) find \( f_{xx} + f_{yy} \).
   
   answer: \( (x^2 + y^2)e^{xy} \)
3. A cylinder initially has radius \( r = 5 \) and height \( h = 8 \), then the radius is increased to 5.1 and the height is decreased to 7.8. Given that the surface area is \( A = 2\pi(r^2 + rh) \), calculate both

a. The exact change in surface area, \( \Delta A \), and

answer: 4.9637

b. The approximate change in surface area, \( dA \).

answer: 5.0265

4. If \( f(x,y) = x^3 - 6xy + y^3 \), find all critical points and classify each as a local minimum, local maximum, or saddle point.

answer: (0, 0) is saddle point, (2, 2) is local minimum.

5. If \( (U_x, U_y, U_z) = (3, 11, -1) \) at the point \((-1, 0, 0)\), which has spherical coordinates \( \rho = 1, \phi = \frac{\pi}{2}, \theta = \pi \), find \( U_\theta \) at this point. For spherical coordinates,

\[
\begin{align*}
x & = \rho \sin(\phi)\cos(\theta) \\
y & = \rho \sin(\phi)\sin(\theta) \\
z & = \rho \cos(\phi)
\end{align*}
\]

answer: \(-11\)