Math 4326 Dr. Duval

LINEAR ALGEBRA Homework

Monday, April 6 – **Revised**

Follow the separate general guidelines for Parts A,B,C. Be sure to include and label *all* four standard parts (a), (b), (c), (d) of Part A in what you hand in.

Orthonormal Bases

Section 6.B

We will only be considering material in this section up to and including the proof of result 6.35 (Orthonormal list extends...)

A: Reading questions. Due by 2pm, Sun., 12 Apr.

- 1. Verify the lists in Example 6.24 are indeed orthonormal, as claimed in the text.
- 2. Demonstrate result 6.30 (Writing a vector...) with $V = \mathbf{F}^3$, orthonormal basis (e_1, e_2, e_3) given by the list in Example 6.24(c), and v = (9, 1, 5).
- 3. Try to read the proof of the Gram-Schmidt Procedure (result 6.31) without worrying too much about the precise algebraic details of the equation defining e_j or the calculation in the middle of p. 183. The second sentence of the statement of the result says, "For j = 2, ..., m, define e_j inductively...". What, in your own words, does that mean in this case?
- 4. Near the bottom of p. 182, the text asks, "does $\mathcal{P}_m(\mathbf{F})$, with inner product [given by 6.4(c)] have an orthonormal basis?" Answer this question, and explain your answer. [Note: you do **not** have to produce such a basis, just decide whether or not it exists.]
- B: Warmup exercises. For you to present in class. Due by the end of class Mon., 13 Apr. Exercises 6.B: 5.

Orthogonal Complements and Minimization Problems Section 6.C

A: Reading questions. Due by 2pm, Tue., 14 Apr.

- 1. Find U^{\perp} for U = span((9, 1, 5)) in $V = \mathbb{R}^3$. Describe U^{\perp} geometrically in this case.
- 2. Verify result 6.47 (Direct sum... orthogonal complement) in the case of question 1 above.
- 3. Find $P_U v$ for v = (1, 2, 3) and U = span((9, 1, 5)) in $V = \mathbb{R}^3$.
- 4. In Example 6.58, approximating $\sin x$ by a 5th-degree polynomial, explain how $\int_{-\pi}^{\pi} |\sin x u(x)|^2 dx$ is minimized using the inner product 6.59 and result 6.56 (Minimizing the distance...).
- B: Warmup exercises. For you to present in class. Due by end of class Wed., 15 Apr. Exercises 6.C: 4, 11.