100%

STAT 3320

Homework 1 (Sect 2.1, Problems 2, 8 Sect 2.2 Problem 12, 14, 18)

(2) right (R), left (L), straight (S).

(a) A = All three vehicles go in the same direction.

= {RRR, LLL, SSS}

4

(b) B = All three vehicles take different directions
= {RLS, RSL, LRS, LSR, SRL, SLR}

4

(c) C = { Exactly two of the three vehicles turn right

= {RRL, RRS, RLR, RSR, LRR, SRR}

4

(d) D = Exactly two vehicles go in the same direction

= {RRL, RRS, RLR, RSR, LRR, SRR, LLR, LLS, SR S, SLS, LSS, RSS, SSR, SSL, LRL, LSL, RLL, SLL,

4

(e) D' = The complement of Exactly two vehicles go in the same direction

= All vehicles go in different direction or all go in same direction.

= {RRR, SSS, LLL, RLS, LRS, RSL, LSR, SRL, SLR}

4

C ∪ D = { D

C ∩ D = C

(3) Let A_i denote the event that the plant at site i is completed by the contract date.

S = Sample space

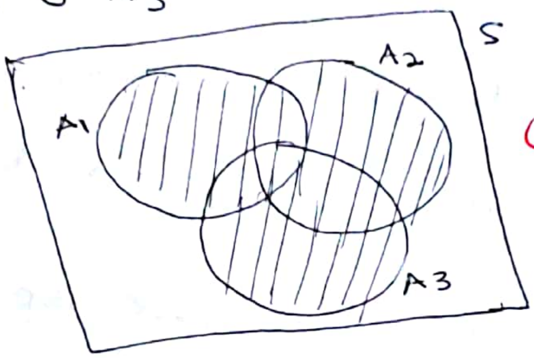
A_1 = the event that the plant at site 1 is completed by the contract date.

A_2 = the event that the plant at site 2 is completed by the contract date.

A_3 = the event that the plant at site 3 is completed by the contract date.

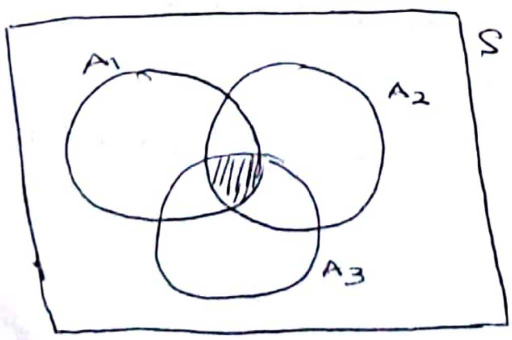
(a) Atleast one plant is completed by the contract date

$= A_1 \cup A_2 \cup A_3$ (1)



(2)

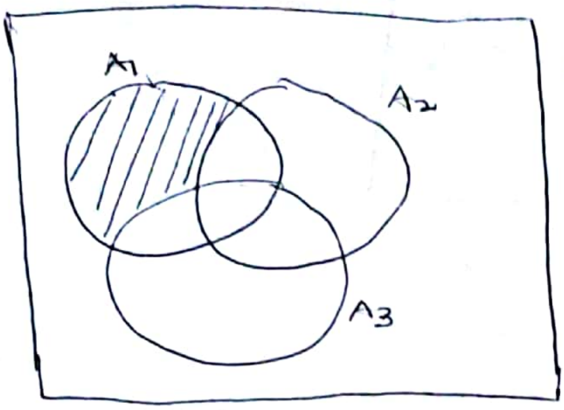
(b) All plants are completed by the contract date which means plant is completed at site 1 and site 2 and site 3. $= A_1 \cap A_2 \cap A_3$ (2)



(2)

ⓒ Only the plant at site 1 is completed by the contract date. which means all possible outcomes except those which include events A_2 and A_3.

$= A_1 \cap A_2' \cap A_3'$ ⑤



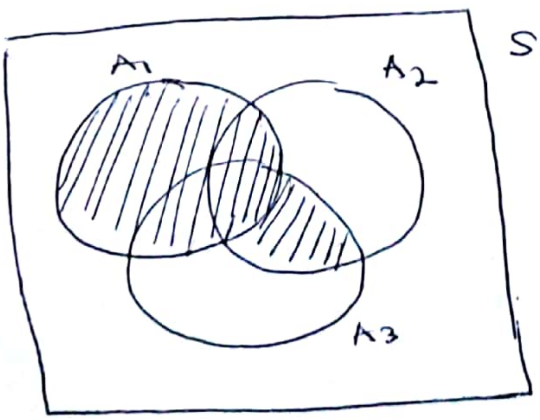
⑤

ⓓ Exactly one plant is completed by the contract date. implying that none of the outcomes that contain more than one event can be include

$= (A_1 \cap A_2' \cap A_3') \cup (A_1' \cap A_2 \cap A_3') \cup (A_1' \cap A_2' \cap A_3)$ ②

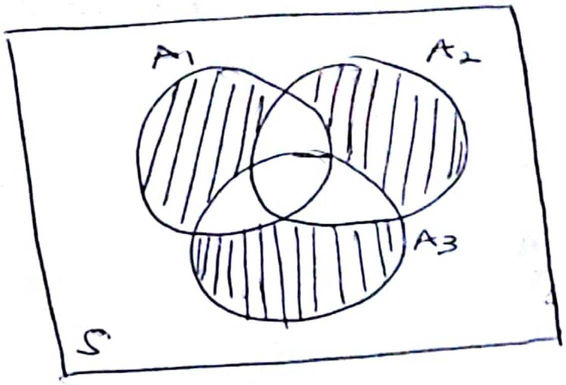
ⓔ Either the plant at site 1 or both of the other two plants are completed by the contract date which is the same as all possible outcomes that are <u>either</u> in event A_1 <u>or</u> in events A_2 <u>and</u> A_1

$= A_1 \cup (A_2 \cap A_3)$ ②



②

d:- Venn diagram for (d)



9th Edition

(12) ~~Consider~~ ... Suppose that $P(A) = 0.6$ and $P(B) = 0.4$

(a) Could it be the case that $P(A \cap B) = 0.5$? Why or why not

Solution

No since $P(A \cap B) = P(B) - \cancel{P(B' \cap A)} \cdot P(B \cap A')$

Hence $P(A \cap B) \leq P(B)$ ~~and $P(A \cap B) < P(B)$~~

4

(b) From now on, suppose that $P(A \cap B) = 0.3$, what is the probability that the selected student has at least one of these two types of cards?

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

4

$$= 0.6 + 0.4 - 0.3$$

$$= 0.7.$$

(c) $$P(A' \cap B') = 1 - P(A \cup B)$$

$$= 1 - 0.7$$

4

$$= 0.3$$

(d) $$P(A \cap B') = P(A) - P(A \cap B)$$

$$= 0.6 - 0.3$$

4

$$= 0.3.$$

(e) $$P(A \cap B') \text{ or } P(A' \cap B) = P(A \cap B') \cup P(A' \cap B)$$

$$= P(A) - P(A \cap B) + P(B) - P(A \cap B$$

$$= P(A) + P(B) - 2P(A \cap B)$$

4

$$= 0.6 + 0.4 - 2(0.3)$$

$$= 0.4$$

(14) Let C denote the event when an adult regularly consumes coffee.

Let S denote the event when an adult regularly consumes carbonated soda.

$p(C) = 0.55$ $\quad p(S) = 0.45$ $\quad p(C \cup S) = 0.70$

(a) $p(C \text{ and } S) = p(C \cap S)$

$= p(C) + p(S) - p(C \cup S)$

$= 0.55 + 0.45 - 0.70$

$= 0.30$

10

(b) p(randomly selected adult does not regularly consume atleast one of these two products)

That is,

$p((C \cup S)') = 1 - p(C \cup S)$

$= 1 - 0.7$

$= 0.3$

10

(17) p(atleast two bills need to be selected to get first $10 bill)

= p(not selecting $10 bills in first selection)

= 1 − [p(selecting $10 bill in first selection)]

$= 1 - \frac{5}{15}$

$= 1 - 0.3333$

$= 0.6667$

20

Scanned by CamScanner

Homework 2 (STAT 3320)

Sect. 2.3, Problems #~~13~~ 50, 56, 60, 66
Sect. 2.4, Problems # ~~13~~

(34) Computer keyboard failures can be attributed to electrical or mechanical defects...

Number of failed keyboards = 25
Failed keyboards due to electrical defects = 6
mechanical = 19

(a) Randomly selecting 5 of these keyboards means selecting any 5 out of the 25 without any order
This results in a combination problem;

$$= \binom{25}{5} = \frac{25!}{(25-5)!\,5!} = \frac{25!}{20!\,5!} = \frac{25\times24\times23\times22\times21\times\cancel{20!}}{\cancel{20!}\times5!}$$

$$= 53{,}130$$

Therefore there are 53,130 ways of selecting any 5 out of the 25.

(b) Randomly selecting so that exactly two have electrical defects implies that choose the two specifically from electrical defects and the rest of the three from mechanical defects. $= \binom{6}{2}$ and $\binom{19}{3} = \binom{6}{2}\times\binom{19}{3}$

$$= \frac{6!}{(6-2)!\,2!}\times\frac{19!}{(19-3)!\,3!}$$

$$= 15\times969$$

$$= 14{,}535 \text{ ways}$$

(c) Probability that ~~the~~ atleast 4 out of the 5 randomly selected keyboards have a mechanical defect implies two scenarios;

Finding the chances that 4 are mechanical and 1 is electrical or finding the chances that all 5 are mechanical defects.

$$= \frac{\binom{19}{4} \times \binom{6}{1}}{\binom{25}{5}} + \frac{\binom{19}{5}\binom{6}{0}}{\binom{25}{5}}$$

$$= \frac{23,256}{53,130} + \frac{11,628}{53,130} = 0.6566$$

50

(50) Short sleeve

Pattern

Size	PL	Pr	St	Total
S	0.04	0.02	0.05	0.11
M	0.08	0.07	0.12	0.27
L	0.03	0.07	0.08	0.18
Total	0.15	0.16	0.25	0.56

Long sleeve

Pattern

Size	P.L	Pr	St	Total
S	0.03	0.02	0.03	0.08
M	0.10	0.05	0.07	0.22
L	0.04	0.02	0.08	0.14
Total	0.17	0.09	0.18	0.44

Let; X be the random variable representing next shirt sold

SS, LS, S, M, L represent short sleeve, long sleeve, Small Medium and Large respectively.

(a) $P(X=M, LS, Pr) = P(X=M \text{ and } X=LS \text{ and } X=Pr)$

$= P(M \cap LS \cap Pr)$

$= 0.05$

We just have to look at the long sleeve table and find the intersection of Pr and M. Note that $P(M \cap LS \cap Pr) \neq P(M) \times P(LS) \times P(Pr)$. {Not independent events.

(b) $P(X=M, Pr) = P(X=M, X=Pr, X=LS) + P(X=M, X=Pr, X=SS)$

$= P(M \cap Pr \cap LS) + P(M \cap Pr \cap SS)$

$= 0.05 + 0.07$

$= 0.12$

(c) $P(X=SS) = P(X=SS \text{ and } X=S) + P(X=SS \text{ and } X=M) + P(X=SS \text{ and } X=L)$

$= P(SS \cap S) + P(SS \cap M) + P(SS \cap L)$

$= 0.11 + 0.27 + 0.18 = 0.56$

$$P(X = LS) = P(X = LS \text{ and } X = S) + P(X = LS \text{ and } X = M) + P(X = LS \text{ and } X = L)$$

$$= P(LS \cap S) + P(LS \cap M) + P(LS \cap L)$$

$$= 0.08 + 0.22 + 0.14$$

$$= 0.44$$

Again the events are dependents so

$P(LS \cap S) \neq P(LS) \times P(S)$ but rather

$P(LS \cap S) = P(LS) \times P(S/LS)$

(d) $P(X = M) = P(M \text{ and } SS) + P(M \text{ and } LS)$

$$= P(M \cap SS) + P(M \cap LS)$$

$$= P(M) \times P(SS/M) + P(M) \times P(LS/M)$$

$$= 0.27 + 0.22$$

$$= 0.49$$

$$P(X = Pr) = P(Pr \text{ and } SS) + P(Pr \text{ and } LS)$$

$$= P(Pr \cap SS) + P(Pr \cap LS)$$

$$= 0.16 + 0.09$$

$$= 0.25$$

(e) $P(X = M \mid \text{Short sleeve } P(\text{Grid})) = P(M \mid SS \cap PL)$

$$= \frac{P(M \cap SS \cap PL)}{P(SS \cap PL)}$$

$$= \frac{0.08}{0.15}$$

$$= 0.53$$

f) $P(SS \mid M \text{ and } PL) = \dfrac{P(SS \cap M \cap PL)}{P(M \cap PL)}$

$= \dfrac{0.08}{0.08 + 0.10}$

$\simeq \dfrac{0.08}{0.18} = 0.44$

$P(LS \mid M \text{ and } PL) = \dfrac{P(LS \cap M \cap PL)}{P(M \cap PL)}$

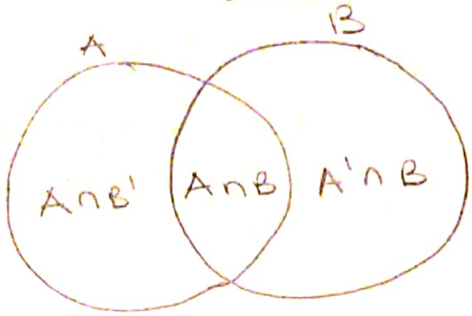
$= \dfrac{0.10}{0.08 + 0.10}$

$= \dfrac{0.10}{0.18}$

$\simeq 0.56$

(56) For any events A and B with $P(B) > 0$,
Show that $P(A|B) + P(A'|B) = 1$

Solution



$$P(A|B) + P(A'|B) = \frac{P(A\cap B)}{P(B)} + \frac{P(A'\cap B)}{P(B)}$$

$$= \frac{P(A\cap B) + P(A'\cap B)}{P(B)}$$

$$= \frac{P(B)}{P(B)}$$

$$= 1$$

(60) Let; D denotes Aircraft discovered
E denotes Emergency locator

$P(D) = 0.70$ $\quad P(E|D) = 0.60$ $\quad P(E'|D') = 0.90$

$P(D') = 0.3$

$$P(E) = P(D)P(E|D) + P(D')P(E|D') = (0.7\times0.6) + (0.3\times0.$$
$$= 0.45$$

If an aircraft has an emergency locator, then the probability that it will not be discovered is;

$$P(D'|E) = \frac{P(D')P(E|D')}{P(E)} = \frac{0.3\times0.1}{0.45} = \frac{1}{15}$$

(b) $P(D|E') = \frac{P(D)\,P(E'|D)}{P(E')} = \frac{0.7 \times [1-P(E|D)]}{1-P(E)}$

$= \frac{0.7 \times (1-0.6)}{1-0.45}$

$= \frac{28}{55}$

(66) Let, E denote event that traveler checks work email

C denote event that traveler uses cell phone

L denote event that a traveler brings a laptop

$P(E) = 0.4$, $P(C) = 0.3$, $P(L) = 0.25$

$P(E \cap C) = 0.23$ $P(E' \cap C' \cap L') = 0.51$

$P(E' \cap C' \cap L') + P(E \cup C \cup L) = 1$

$P(E \cup C \cup L) = 1 - 0.51 = 0.49$

$P(E|L) = 0.88$ $P(L|C) = 0.7$

(a) $P(C|E) = \frac{P(C \cap E)}{P(E)} = \frac{0.23}{0.4} = 0.575$

(b) $P(C|L) = \frac{P(C \cap L)}{P(L)} = \frac{P(L|C)\,P(C)}{P(L)} = \frac{0.7 \times 0.3}{0.25}$

$= 0.84$

(c) $P(C \mid E \cap L) = \dfrac{P(C \cap E \cap L)}{P(E \cap L)}$

$= \dfrac{P(C \cap E \cap L)}{P(E \mid L)\, P(L)}$

$P(C \cap E \cap L) = P(C \cup E \cup L) - P(E) - P(C) - P(L) + P(E \cap L) + P(C \cap L)$
$+ P(E \cap C)$

$=$

$P(C \cap L) = P(L \mid C)\, P(C) = 0.21$

$P(E \mid L)\, P(L) = 0.88 \times 0.25 = 0.22$

$P(C \cap E \cap L) = 0.49 - 0.4 - 0.3 - 0.25 + 0.23 + 0.22$
$+ 0.21$
$= 0.2$

$P(C \mid E \cap L) = \dfrac{0.2}{0.22} = 0.909$

STAT 3320

HW3

Sect 2.5 ; Problems # 74, 76, 82

Supp Ex, # 104, 110

(74) The proportions of blood phenotypes in U.S population are as follows:

$P(A) = 0.40$, $P(B) = 0.11$ $P(AB) = 0.04$ $P(O) = 0.45$

Two randomly selected individuals are independent implies

$$P(O \cap O) = P(O) \times P(O)$$
$$= 0.45 \times 0.45$$
$$= \boxed{0.2025}$$

Two randomly selected have their phenotypes matching

$$P(\text{matching phenotypes}) = P(A \cap A) + P(B \cap B) + P(AB \cap AB) + P(O \cap O)$$
$$= P(A) \cdot P(A) + P(B) \cdot P(B) + P(AB) \cdot P(AB) + P(O) P(O)$$
$$= 0.40^2 + 0.11^2 + 0.04^2 + 0.45^2$$
$$= \boxed{0.3762}$$

(76) Let X be the number of errors that occur in a division where each division is independent

P(at least 1 error in 1 billion divisions) $= P(X \geq 1) = P(X=1) + P(X=2) + \cdots$

$= 1 - P(X=0)$

$= 1 - \left(1 - \frac{1}{9 \times 10^9}\right)^{10^9}$

$= 1 - 0.8948$

$= \boxed{0.1052}$

(82) Let, A be the ~~event that the red die shows 3 dots, B be the event that the green die shows 4 dots, and C be the event that the total number of showing on the two dice is 7~~

~~$S = \{($~~

~~$A = \{(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)\}$~~

~~$P(A) = \frac{n(A)}{n(S)}$~~

(32) Two fair dice are rolled, red (R) and green (G)

R \ G	1	2	3	4	5	6	
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)	C
2	(2,1)			(2,4)	(2,5)		
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)	A
4			(4,3)	(4,4)			
5		(5,2)		(5,4)			
6	(6,1)			(6,4)			
				B			

Let S be the sample space of the experiment

$S = \{(1,1), (1,2), \ldots, (2,1), (2,2), \ldots (6,1), (6,2), \ldots (6,6)\}$

$n(S) = 36$

A is the event that the red die shows 3 dots

$A = \{(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)\}$

$n(A) = 6$

B is the event that the ~~total number of dots~~ green die shows 4

$B = \{(1,4), (2,4), (3,4), (4,4), (5,4), (6,4)\}$

$n(B) = 6$

C is the event that the total number of dots showing on the two dice is $7 = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$. $n(C) = 6$.

$P(A) = \dfrac{n(A)}{n(S)} = \dfrac{6}{36} = \dfrac{1}{6}$; $P(B) = \dfrac{6}{36} = \dfrac{1}{6}$, $P(C) = \dfrac{n(C)}{n(S)} = \dfrac{1}{6}$

Now we show that the three events are pairwise independent;

~~$P(A \cap B) =$~~ $A \cap B = \{(3,4)\}$

$P(A \cap B) = \dfrac{1}{36} = \dfrac{n(A \cap B)}{n(S)} = P(A) \times P(B) = \dfrac{1}{6} \times \dfrac{1}{6} = \dfrac{1}{36}$

$A \cap C = \{(3,4)\}$

$P(A \cap C) = \dfrac{n(A \cap C)}{n(S)} = \dfrac{1}{36} = P(A) \times P(C) = \dfrac{1}{6} \times \dfrac{1}{6} = \dfrac{1}{36}$

$B \cap C = \{(3,4)\}$

$P(B \cap C) = \dfrac{n(B \cap C)}{n(S)} = \dfrac{1}{36} = P(B) * P(C) = \dfrac{1}{6} \times \dfrac{1}{6} = \dfrac{1}{36}$.

Hence they are pairwise independent since all three pairs satisfy the condition.

For mutual independence, we are to show that $P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$?

$A \cap B \cap C = \{(3,4)\}$.

$$P(A \cap B \cap C) = \frac{n(A \cap B \cap C)}{n(S)} = \frac{1}{36}$$

$$P(A) \times P(B) \times P(C) = \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{6^3} = \frac{1}{216}$$

Since $P(A \cap B \cap C) \neq P(A)P(B)P(C)$;

we conclude that the events are not mutually independent.

104 Let D denote the event that a component is defective. Now the probability of this event is;

$$P(D) = P(D \cap A_1) + P(D \cap A_2) + P(D \cap A_3)$$

$$= P(D|A_1) \cdot P(A_1) + P(D|A_2) \cdot P(A_2) + P(D|A_3) \cdot P(A_3)$$

$$= 0.05(0.5) + 0.08(0.3) + 0.1(0.2)$$

$$= \boxed{0.069}$$

If a randomly selected component needs rework, that means it is defective

$$P(A_1|D) = \frac{P(A_1 \cap D)}{P(D)} = \frac{P(D|A_1) \cdot P(A_1)}{P(D)} = \frac{0.05 \times 0.5}{0.069} = \boxed{0.362}$$

$$P(A_2|D) = \frac{P(A_2 \cap D)}{P(D)} = \frac{P(D|A_2) P(A_2)}{P(D)} = \frac{0.08(0.3)}{0.069} = \boxed{0.348}$$

The probability that a company was assembled at A_2 given that it is defective is $P(A_2|D) = 0.348$

$$P(A_3|D) = \frac{P(A_3 \cap D)}{P(D)} = \frac{P(D|A_3) P(A_3)}{P(D)} = \frac{0.1 \times 0.2}{0.069} = \boxed{0.290}$$

Scanned by CamScanner

(110) A denote the event that the New York flight is full and define events B and C analogously for ~~B~~ Atlanta and Los Angeles.

$P(A) = 0.9$; $P(B) = 0.7$; $P(C) = 0.8$

(a) P(all three flights are full) = $P(A \cap B \cap C)$

Since the events are independent, we have

$$P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$$
$$= 0.9 \times 0.7 \times 0.8$$
$$= \boxed{\cancel{0.12}} \; \boxed{0.504}$$

(b) P(at least one flight is not full)

= 1 - P(all three flights are full

$$= 1 - \overset{0.504}{\cancel{0.12}} = \boxed{\cancel{0.88}} \; \boxed{0.496}$$

(c) P(only New York flight is full) = $P(A \cap B' \cap C')$

$$= P(A) \times P(B') \times P(C')$$
$$= \cancel{0.9 \times 0.7 \times 0.8} \; 0.9 \times 0.3 \times 0.2$$
$$= \boxed{\cancel{0.18}} \; \boxed{0.054}$$

(d) P(exactly one of the three flights is full)

= P(Only New York) + P(Only Atlanta) + P(Only Los Angeles)

$= P(A \cap B' \cap C') + P(A' \cap B \cap C') + P(A' \cap B' \cap C)$

$= P(A) \times P(B') \times P(C') + P(A') \times P(B) \times P(C') + P(A') \times P(B') \times P(C)$

$= (0.9 \times 0.3 \times 0.2) + (0.1 \times 0.7 \times 0.2) + (0.1 \times 0.3 \times 0.8)$

$= 0.054 + 0.014 + 0.024$

$= \boxed{0.092}$