Some R problems for practice:

1. Generate a sequence of 100 numbers between 1 and 10. Call the sequence x. Produce a line plot of x in reverse order.

2. Create the vectors of:

   (a) all integers from 1 to 20.

   (b) all multiples of 3 from 1 to 100.

   (c) all integers from 1 to 100, which are not multple of 5.

   (d) (4, 6, 3, 4, 6, 3, . . . , 4, 6, 3) where there are 10 occurrences of 4.

   (e) (4, 4, . . . , 4, 6, 6, . . . , 6, 3, 3, . . . , 3) where there are 10 occurrences of 4, 20 occurrences of 6 and 30 occurrences of 3.

   (f) the numbers $\left(3, \frac{3^2}{2}, \frac{3^3}{3}, \ldots, \frac{3^{20}}{20}\right)$

3. Calculate the following

   (a) $\sum_{i=20}^{90}(i^4 + 4i^5)$

   (b) $\sum_{i=1}^{10}(\frac{2^i}{i} + \frac{3^i}{i^3})$

4. Execute the following lines which create two vectors of random integers which are chosen with replacement from the integers 0, 1, . . . , 999. Both vectors have length 250.

```
set.seed(50)
xVec <- sample(0:999, 250, replace=T)
yVec <- sample(0:999, 250, replace=T)
```

   Suppose $x = (x_1, x_2, \ldots, x_n)$ denotes the vector xVec and $y = (y_1, y_2, \ldots, y_n)$ denotes the vector yVec.

   (a) Create the vector $(y_2 - x_1, \ldots, y_n - x_{n-1})$.

   (b) Create the vector $\left(\frac{sin(y_1)}{cos(x_2)}, \frac{sin(y_2)}{cos(x_3)}, \ldots, \frac{sin(y_{n-1})}{cos(x_n)}\right)$

   (c) Create the vector $(x_1 + 2x_2 - x_3, x_2 + 2x_3 - x_4, \ldots, x_{n-2} + 2x_{n-1} - x_n)$.

5. This question uses the vectors xVec and yVec created in the previous question.

   (a) Pick out the values in yVec which are $>$ 600.

   (b) What are the index positions in yVec of the values which are $>$ 600?

   (c) What are the values in xVec which correspond to the values in yVec which are $>$ 600? (By correspond, we mean at the same index positions.)

   (d) How many values in yVec are within 200 of the maximum value of the terms in yVec?

(e) Sort the numbers in the vector xVec in the order of increasing values in yVec.

(f) Pick out the elements in yVec at index positions 1, 4, 7, 10, 13, . . .

6. Suppose

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$$

(a) Check that $\mathbf{A}^3 = \mathbf{0}$ (matrix multiplication) where $\mathbf{0}$ is a 3×3 matrix of 0's.

(b) Replace the third column of $\mathbf{A}$ by the sum of the second and third columns.

7. Calculate (using *for* loop)

(a) $\sum_{i=1}^{20} \sum_{j=1}^{5} \frac{i^4}{(3+j)}$

(b) $\sum_{i=1}^{20} \sum_{j=1}^{5} \frac{i^4}{(3+ij)}$

(c) $\sum_{i=1}^{20} \sum_{j=1}^{i} \frac{i^4}{(3+j)}$

8. Produce matrix plot of a matrix which has 10 rows and has as elements all the numbers divisible by 5 in between 1 and 200.

9. Compute the mean, median, standard deviation and 82nd quantile of all the numbers in between 1 and 50 which are divisible by 2.25.

10. Create a 6×10 matrix of random integers chosen from 1,2,...,10 with setting seed at 75.

(a) Find the number of entries in each row which are greater than 4.

(b) Which rows contain exactly two occurrences of the number seven?

(c) Find those pairs of columns whose total (over both columns) is greater than 75.

11. Write functions tmpFn1 and tmpFn2 such that if xVec is the vector $(x_1, x_2, \ldots, x_n)$, then tmpFn1(xVec) returns the vector $(x_1, x_2^2, \ldots, x_n^n)$ and tmpFn2(xVec) returns the vector $(x_1, \frac{x_2^2}{2}, \ldots, \frac{x_n^n}{n})$.

12. Write a function func which takes 2 arguments x and n where x is a single number and n is a strictly positive integer. The function should return the value of

$$1 + x + \frac{x^2}{2} + \ldots + \frac{x^n}{n}.$$

13. Consider the continuous function

$$f(x) = \begin{cases} x^2 + 2x + 3 & \text{if } x < 0 \\ x + 3 & \text{if } 0 \le x < 2 \\ x^2 + 4x - 7 & \text{if } 2 \le x. \end{cases}$$

Write a function tmpFn which takes a single argument xVec. The function should return the vector of values of the function $f(x)$ evaluated at the values in xVec. Hence plot the function $f(x)$ for $-3 < x < 3$.

14. Write a function which takes a single argument which is a matrix. The function should return a matrix which is the same as the first matrix but every odd number of the first matrix is doubled.

15. Write a function which takes 2 arguments n and k which are positive integers. It should return the $n \times n$ matrix:

$$\begin{bmatrix} k & 1 & 0 & 0 & \dots & 0 & 0 \\ 1 & k & 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & k & 1 & \dots & 0 & 0 \\ 0 & 0 & 1 & k & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & 1 & k \end{bmatrix}.$$

Print the output of the function with $n = 5$ and $k = 2$.

16. Suppose $x_0 = 1$ and $x_1 = 2$ and

$$x_j = x_{j-1} + \frac{2}{x_{j-1}} \quad \text{for } j = 1, 2, \dots$$

Write a function testLoop which takes the single argument n and returns the first $n-1$ values of the sequence $\{x_j\}_{j\geq 0}$.

17. Given a vector $xVec = (x_1, \dots, x_n)$, the sample autocorrelation of lag k is defined to be

$$r_k = \frac{\sum_{i=k+1}^{n}(x_i - \bar{x})(x_{i-k} - \bar{x})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}.$$

Write a function tmpFn(xVec) which takes two arguments k and $xVec$ which is a vector, and returns the list of two values: k and r_k.

18. Suppose $z_1, z_2, \dots, z_n$ is a time series. Then we define the exponentially weighted moving average of this time series as follows: select a starting value m_0 and select a discount factor δ. Then calculate $m_1, m_2, \dots, m_n$ as follows: for $t = 1, 2, \dots, n$

$$e_t = z_t - m_{t-1}$$

$$m_t = m_{t-1} + (1 - \delta)e_t.$$

Write a function tsEwma(tsDat, m0=0, delta=0.7) where tsDat is a time series, m0 is the starting value m_0 and delta is δ. The function should return $m_1, m_2, \dots, m_n$ in the form of a time series (ts object) with start=c(1960,3) and frequency=12.

19. Write a function, called myListFn, which takes a single argument n and implements the following algorithm:

- Simulate n independent numbers, denoted $\mathbf{x} = (x_1, \dots, x_n)$, from $N(0, 1)$ distribution.
- Calculate the mean $\bar{x}$.

- if $\bar{x} \geq 0$, then simulate n independent numbers, denoted $\mathbf{y} = (y_1, \ldots, y_n)$, from the exponential distribution with mean $\bar{x}$. [Hint. Use function rexp. mean=1/rate.] if $\bar{x} < 0$, then simulate n independent numbers, denoted $\mathbf{z} = (z_1, \ldots, z_n)$, from the exponential density with mean $-\bar{x}$. Set $\mathbf{y} = -\mathbf{z}$.
- Calculate k which is the number of j with $|y_j| > |x_j|$.
- Return the list of $\mathbf{x}$, $\mathbf{y}$ and k with names xVec, yVec and count respectively.

20. Consider the following matrix.

$$A = \begin{bmatrix} 1 & 2 & 6 \\ 4 & 3 & 0 \\ 5 & 1 & 9 \end{bmatrix}.$$

If you draw image of A using image(A) in R, the first, second and third column of A will be shown respectively in the bottom, middle and top row of the image. Create a matrix B by adjusting A in such a way that the first, second and third column of image(B) will show the first, second and third column of A.