Math 4329, Test I

Name _____

1. a. If $f(x) = 2x^4 + x^3$, find the Taylor polynomial $T_3(x)$ of degree 3
    which matches f, f', f'' and f''' at $a = 1$.

    b. Use the Taylor remainder formula to get a reasonable bound on
    the error $|f(x) - T_3(x)|$ in the interval $-1 \le x \le 3$.

2. Write the quadratic formula root $[-b + \sqrt{b^2 - 4ac}]/(2a)$ in a form so that
   there are no serious problems with roundoff error, when b is positive
   and very large compared to ac.

3. If Newton's method is used to find a root of $f(x) \equiv (x-3)^5 = 0$, for
   what range of starting values x_0 will we get convergence to the root
   $r = 3$? What is the order of convergence of Newton's method in this
   case?

4. If $a = -10, b = 10$ and $f(a)$ and $f(b)$ have opposite signs, about how many bisection iterations are required to find a root between a and b to an accuracy of 10^{-10}?

5. Compute the experimental order of convergence for a root finder with errors in 3 consecutive iterations of $10^{-4}, 10^{-7}$ and 10^{-14}.

6. The fixed point iteration $x_{n+1} = x_n + sin(x_n)$ has roots at $r = n\pi$ for any integer n. For which of these roots (which values of n) will the interation converge? What will be the order of convergence for these roots?