Math 4329, Test II

Name _____

1. If

$$\begin{array}{rcll} s(x) & = & 0 & for\ 1 \leq x \leq 2 \\ s(x) & = & A(x-2)^3 & for\ 2 \leq x \leq 3 \end{array}$$

   a. For what value(s) of A is $s(x)$ a cubic spline?

   b. For what value(s) of A is $s(x)$ a natural cubic spline?

2. If $P_4(x)$ is the fourth degree polynomial that interpolates to $f(x) = sin(2x)$ at $x = 0, 0.1, 0.2, 0.3, 0.4$, find a reasonable bound on the error at $x = 0.35$.

3. Use Taylor series expansions to determine the error in the approximation $u'''(t) \approx \frac{u(t+3h)-3u(t+2h)+3u(t+h)-u(t)}{h^3}$

4. Find A, B which make the approximation

   $\int_0^h f(x)dx \approx Ahf(0) + Bhf(\frac{2h}{3})$

   as high degree of precision as possible. With your choice of A, B, what is the degree of precision, and what is the order of the error (power of h that the global error is proportional to) in this approximation?

5. Consider the linear system:

$$\begin{bmatrix} 7 & 2 & 1 \\ 0 & 3 & 2 \\ 1 & -3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ -3 \end{bmatrix}$$

a. Write out the equations for the Jacobi iterative method for solving this system (don't actually do any iterations).

b. Write out the equations for the Gauss-Seidel iterative method for solving this system.

c. True or False: the Jacobi iterative method (5a) will converge for *any* starting vector (x_0, y_0, z_0). Give a reason for your answer.

d. Given that

$$A^{-1} = \begin{bmatrix} 0.1419 & -0.0878 & 0.0068 \\ 0.0135 & 0.2297 & -0.0946 \\ -0.0203 & 0.1554 & 0.1419 \end{bmatrix}$$

find the condition number of A (using L_∞ norm). If you were to solve the linear system above using Gaussian elimination with partial pivoting, would you expect serious roundoff errors?