Math 5330, Test II

Name ______________________________

1. Given that the QR decomposition of A is

$$Q = \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \\ 0 & \frac{1}{\sqrt{3}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \end{bmatrix}, R = \begin{bmatrix} 1 & -2 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

use this to find x which minimizes $\|Ax - b\|_2$, where $b = (2, 3, 1)$.

2. a. Prove that if z is a solution to $AA^Tz = b$, then $x \equiv A^Tz$ is a solution of $Ax = b$ of minimum norm.

b. Use (a) to find the quadratic polynomial $a + bx + cx^2$, with minimum value of $a^2 + b^2 + c^2$, which passes through the two points $(0, 1), (1, 3)$.

3. If

$$A = \begin{bmatrix} 12 & 5 \\ 5 & 12 \end{bmatrix}$$

a. Do one QR iteration on A.

b. Do one LR iteration on A.

c. Use the Jacobi method to find all eigenvalues and eigenvectors of A. (Note: only one iteration is necessary!)

d. Use the power method to find the largest eigenvalue (in absolute value) of A, starting with $x_0 = (1, 2)$.

4. Find an orthogonal matrix Q such that QAQ^{-1} is tridiagonal, if

$$A = \begin{bmatrix} 2 & 5 & -12 \\ 5 & 1 & 7 \\ -12 & 7 & 1 \end{bmatrix}$$