Math 5330, Test II

Name ______________________________

1. Given that the QR decomposition of A is
$$Q = \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \\ 0 & \frac{1}{\sqrt{3}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \end{bmatrix}, R = \begin{bmatrix} 2 & 5 \\ 0 & -3 \\ 0 & 0 \end{bmatrix}$$
use this to find x which minimizes $\|Ax-b\|_2$, where $b = (\sqrt{3}, -\sqrt{3}, \sqrt{3})$.

2. Prove that if z is a solution to $AA^Tz = b$, then $x \equiv A^Tz$ is a solution of $Ax = b$ of minimum norm.

3. Find all eigenvalues of the pseudo-triangular matrix
$$\begin{bmatrix} 1 & 3 & 8 & -1 \\ -4 & 2 & 3 & 4 \\ 0 & 0 & 2 & 7 \\ 0 & 0 & 0 & 11 \end{bmatrix}$$

4. If the Jacobi iteration $A_{n+1} = Q_n^T A_n Q_n$, where $A_1 = A$ converges to diagonal form in, say, 5 iterations, so that $A_6 \approx D$, what are the eigenvalues of A, and what are the eigenvectors?

5. If

$$A = \begin{bmatrix} 2 & 12 & -5 \\ 12 & 1 & 7 \\ -5 & 7 & 1 \end{bmatrix}$$

a. Find an orthogonal matrix Q such that QAQ^{-1} is upper Hessenberg.

b. Find an elementary matrix M such that MAM^{-1} is upper Hessenberg.

6. Do one complete iteration of the LR method, starting with

$$A = \begin{bmatrix} 1 & 1 & 0 \\ -3 & -2 & 1 \\ 0 & 4 & -8 \end{bmatrix}$$

7. Use the power method to find the largest (in absolute value) eigenvalue of

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 10 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Start with $(1, 5, 1)$ and do 3 iterations. What is the corresponding eigenvector?